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GEORGINA LOWELL PUTNAM

OF BOSTON

Received, July 1, 1914.

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A

KEY,

CONTAINING THE

STATEMENTS AND SOLUTIONS OF QUESTIONS

IN

DAVIES' ELEMENTARY ALGEBRA;

FOR

THE USE OF TEACHERS ONLY.

NEW YORK:
PUBLISHED BY A. S. BARNES & CO.,
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PREFACE.

THE question, whether a Key to a work on mathematics facilitates the acquisition of knowledge, is one about which there is much diversity of opinion. If the business of teaching were pursued as a profession — if the teachers in our schools and seminaries looked to no other employment, and gave their entire thoughts and time to the business of instruction, they would have abundant means to prepare, in the best manner, all the exercises for their pupils.

But, as yet, the case is far different. Teaching, with most instructers, is an occasional and temporary business, and not a permanent profession. Engaged, generally, in preparing themselves for other pursuits, and at the same time giving instruction in various branches of education, they have neither the time nor opportunity for that careful preparation which is needed, and must, therefore, avail themselves of all the aids which they can command.

It was not intended, originally, to prepare a Key to the Elementary Algebra, but the urgent request of many teachers has changed that determination.

It was not thought best to work out the simple examples which are given as illustrations, nor those which are given to perfect the scholar in the mechanical part of algebra; and hence the work in the Key is limited to the questions only These alone, it was supposed, presented difficulties in the statements, which are fully given, leaving the solution of the equations to be made by the pupil. This will obviate much of the misuse to which a Key may be applied, should it chance to fall into the hands of the student.

The large figures at the head of each page point out the corresponding page of the Algebra

KEY

TO

DAVIES' ELEMENTARY ALGEBRA

(Page 111.)

(13.)

Denote D's share by x. Then, by the conditions of the question,

$x+360=$ B's share,

and $2x+720-1000=$ A's share:

but $x=$ D's share,

and $360=$ C's share;

hence $4x+440=2520$, the whole estate, from which equation we find $x=520$.

(14.)

Let $x=$ the share of each daughter. Then, by the conditions of the question,

$2x=$ the share of each son.

Also, since there are three daughters and two sons,

$3x=$ the amount received by the daughters,

and $4x=$ the amount received by the sons;

Also, $7x+500=$ the amount received by the widow;
and $14x+500=7500$, the whole estate; and from this last equation the value of x is readily found, equal to 500

(15.)

Let $x=$ the number of women. Then, by the conditions of the question,

$x+8=$ the number of men,
and $2x+8+20=$ children:
but $x\ \ =$ women; hence
$4x+36=180$, the whole number; from which we find $x=36$.

(16.)

Let $x=$ the share of the youngest brother.
Then $x+40=$ 2d son's share,
$x+80=$ 3d son's share,
$x+120=$ 4th son's share,
$x+160=$ 5th son's share,
and $5x+400=2000$, the whole estate; from which we find $x=320$.

(17.)

Let the share of A be denoted by x. Then, since A's share is to be to B's as 6 to 11, it follows that B's share will be $\frac{11}{6}$ of A's Hence

$$\frac{11}{6}x= \text{B's share,}$$

and $x+\frac{11}{6}x+300=$ C's share.

Hence $2x+\frac{22}{6}x+300=2850$, the whole estate; from which equation we find $x=450$.

(18.)

Let x denote the number of paces taken by the first person, from the time of starting till the distance between them is 300 feet. Then, the number of paces taken by the second will be represented by $5x$. But since the paces of the first are 3 feet, and those of the second $1\frac{1}{2}$ or $\frac{3}{2}$ feet, the distances travelled will be

$3x=$ the *distance* travelled by first,

and $\frac{3}{2}\times 5x=\frac{15}{2}x=$ the *distance* travelled by second,

hence $\frac{15}{2}x-3x=300$, their distance apart; from which we find $x=66\frac{2}{3}$; that is, the person who steps the longest will have made $66\frac{2}{3}$ paces; and since each pace is 3 feet, he will have travelled

$$66\tfrac{2}{3}\times 3=200 \text{ feet.}$$

If, instead of subtracting $3x$ from $\frac{15}{3}x$, we had written the equation

$$3x-\frac{15}{2}x=300,$$

we should have found

$$x=-66\tfrac{2}{3},$$

which would have shown that the second person travelled farther than the first; which is indeed proved when the distance travelled by the second, minus the distance travelled by the first, is positive.

(19.)

Let x denote the number of days which they worked.

Then, $2x=$ the number of dollars earned by carpenters,

$\frac{24x}{2}=12x$, the amount earned by journeymen,

$\frac{8}{4}x=2x$, the amount earned by the apprentices.

Hence, $16x=144$, the whole sum earned; whence we find $x=9$.

(20.)

Let the sum at interest be denoted by x.

Then, $\frac{4}{5}x=$ what bears an interest of 4 per cent.,

and $\frac{1}{5}x=$ what draws 5 per cent.

Then, since the interest which accrues on any sum for a year, is equal to the sum, multiplied by the rate, divided by 100, we shall have

$\frac{4}{5}x\times\frac{4}{100}=\frac{4}{125}x$, what the first produced.

and $\frac{1}{5}x\times\frac{5}{100}=\frac{1}{100}x$, what the second produced.

Then $$\frac{4}{125}x+\frac{1}{100}x=2940,$$

or $$400x+125x=36750000, \text{ or } x=70000.$$

(21.)

There are several ways in which this example may be solved—

First. We see by the conditions, that the first cock will discharge one gallon in a minute, the second cock half a gallon in a minute, and the third cock one-third of a gallon in a minute. Hence, the quantity discharged by the three cocks in a minute, is equal to

$$1+\frac{1}{2}+\frac{1}{3}=\frac{11}{6} \text{ gallons.}$$

Then, $\frac{11}{6}$ gals. : 60 gals. : : 1 m. : $32\frac{8}{11}$ minutes.

But, to resolve the question in a general way, without regard to the contents of the cask, let x denote the part of the cask which would be emptied in a single minute. Then, as the first cock would empty $\frac{1}{60}$ of the cask, the second $\frac{1}{120}$ of it, and the third $\frac{1}{180}$ of it, in one minute, we have

$$x=\frac{1}{60}+\frac{1}{120}+\frac{1}{180}$$

or $$x=\frac{6}{360}+\frac{3}{360}+\frac{2}{360}$$

or $$x=\frac{11}{360}$$

Now, as $\frac{11}{360}$ of the cask is emptied in one minute, it is evident that to empty the entire cask will require as many minutes as the number 1 contains $\frac{11}{360}$; that is, $32\frac{8}{11}$.

(22.)

Let the number of trees in the orchard be denoted by x

Then $\frac{1}{2}x=$ apple trees,

$\frac{1}{4}x=$ peach trees,

$\frac{1}{6}x=$ plum trees,

$120=$ cherry trees,

$80=$ pear trees,

and $$x=\frac{1}{2}x+\frac{1}{4}x+\frac{1}{6}x+120+80$$

from which we find $x=2400$.

(23.)

Let the number of sheep be denoted by x. Then, by observing the statement of the last problem, we have

$$x=\frac{1}{4}x+\frac{1}{6}x+\frac{1}{8}x+\frac{1}{12}x+450$$

which gives $x=1200$

(24.)

Let $x=$ the value of the horse.

Then $\frac{x}{10}=$ the value of the saddle, and $x+\frac{x}{10}=132,$

which gives $x=120.$

(25.)

Let x denote the amount of rent last year.

Then, $\frac{8x}{100}$ equal the *excess* of the present over the past year, and the rent of the present year will be expressed by

$$x+\frac{8x}{100}=1890,$$

which gives $x=1750.$

(26.)

Let $x=$ the number.

Then, $x-5=$ the difference, and

$$\frac{2}{3}\times(x-5)=40,$$

or $2x-10=120$, or $x=65.$

(27.)

Let $x=$ the length of the post.

Then, $\frac{1}{4}x+\frac{1}{3}x+10=x$

from which we have $x=24.$

(28.)

Let $x=$ the number.

Then, $x-\frac{1}{4}x-\frac{1}{5}x=66,$

from which we have $x=120.$

(29.)

Let the number of beggars be denoted by x.

Then, by the conditions,

$3x-8=$ the amount of money he had,

also, $2x+3=$ the amount of money.

Hence, $3x-8=2x+3$, or $x=11$.

(30.)

Let $x=$ the amount of money which he had.

Then, $\frac{1}{4}x=$ what he first lost,

and $x-\frac{1}{4}x+3=$ what he had left after borrowing.

Then, $\dfrac{\frac{3}{4}x+3}{3}=\dfrac{3x+12}{12}=$ what he lost the second time,

and this taken from $\frac{3}{4}x+3$, or what he had at the commencement of the second game, will give what he had left: that is,

$$\frac{3}{4}x+3-\frac{3x+12}{12}=12;$$

or,

$$\frac{3x+12}{4}-\frac{3x+12}{12}=12;$$

from which we find $x=20$

(31.)

Let x denote the amount laid out by each.

Then, $x+126=$ what A had, after gaining,

and $x-87=$ what B had, after losing.
Then, $x+126=2(x-87)=2x-174$,
from which we find $x=300$.

(32.)

Let x denote the sum which he had at first.
Then $x-2=$ what he had after spending,
$\underline{\quad 2}$
$2x-4=$ what he had after borrowing,
and $2x-6=$ what he had after spending at second tavern,
$\underline{\quad\ 2}$
$4x-12=$ what he had after borrowing,
and $4x-14=$ what he had after spending at third tavern,
$\underline{\quad\ 2}$
$8x-28$, what he had after borrowing, and
then, $8x-30s.=0$.
Hence, $x=3\frac{6}{8}s.=3s.\ 9d.$

[Page 96.]

(11.)

Let the money of A be denoted by x, and that of B by y.
Then, by the first condition, $x+40=5(y-40)=5y-200$;
by the second, $x+y=120$;
from which we readily find $x=60$ and $y=60$.

(12.)

Let x = the father's ago and y that of the son.

Then the father's age, twenty years before, would be represented by $x-20$, and that of the son by $y-20$. Hence,
by the first condition, $x-20=4(y-20)=4y-80$,
by the second, $x=2y$,
from which we find x and y.

(13.)

Let x = what the elder had, and y = what the younger had from the father. Then,
by the first condition $x-\frac{1}{4}x=y+1000$;
then, by second condition, $x-\frac{1}{4}x+2000=2(y+1000-500)$
or $4x-x+8000=8y+8000-4000$,
or $3x=8y-4000$,
from which the value of x and y are easily found.

(14.)

Let x= what John, and y= what Charles had.
Then, $x-15=y+15$,
and $x+15=15(y-15)-10$,
from which we have x and y.

(15.)

Let x= A's salary, and y= B's.
Then, $x+y=900$;
by second condition, $x-\frac{1}{10}x=y+\frac{1}{10}x$;
from which we obtain x and y.

[Page 100.]

(12.)

Let $x=$ the value of the first, and $y=$ that of the second.
Then, by the first condition, $x+7=3y$,
and, by the second condition, $5x=y+7$;
from which we have x and y.

(13.)

Let the numbers be denoted by x and y.
Then, by 1st condition, $6x=5y$,
and by the 2d, $x-1=y-2$;
from which we have $x=5$ and $y=6$.

(14.)

Let the numbers be denoted by x and y.

By 1st condition, $x+2=3\frac{1}{4}y$;

or, $x+2=\frac{13}{4}y$;

By 2d condition, $\frac{x}{2}=y+4$,

from which we find $x=24$ and $y=8$.

(15.)

Let the present ages of the father and son be denoted by x and y. Then,
by 1st condition, $x-12=2y$,

by 2d condition, $4(y-12)+12=x+12$;
from which we have $x=72$ and $y=30$.

[Page 106.]

(4.)

Let x and y denote the numbers.

Then $x-y=7$,
and $x+y=33$;
which equations give $x=20$, and $y=13$.

(5.)

Let $x=$ the greater and $y=$ the lesser part.

Then, by 1st condition, $x+y=75$,
and by 2d condition, $3x=7y+15$;
which give $x=54$, and $y=21$

(6.)

Let $x=$ the wine, and $y=$ the cider.

Then, by 1st condition, $\frac{x+y}{2}+25=x$;
and by 2d condition, $\frac{x+y}{3}-5=y$;
from which we have $x=85$, and $y=35$;

[Page 107.]

(7.)

Let $x=$ the number of guineas, and $y=$ the number of moidores used

Now it is evident that the number of pieces used, of each kind, multiplied by the number of shillings in the piece, will give the number of shillings paid in that particular kind of money. That is, $21x$ will be the number of shillings paid in guineas, and $27y$ the number paid in moidores. Then observing that the whole bill £120=2400*s.*, we have,

by 1st condition, $x+y=100$,
by 2d condition, $21x+27y=2400$;
which give $x=50$, and $y=50$.

(8.)

Let $x=$ the distance travelled by the first, and $y=$ the distance travelled by the second;

Then $x+y=150$.

But since the first travels 8 miles, while the second travels but 7, the distance which they respectively travel will be in the proportion of 8 to 7: that is

$$x : y :: 8 : 7;$$

or $7x=8y$;

from which we find $x=80$, and $y=70$; and if the entire distance travelled by each be divided by the distance travelled each day, the quotient will be the time, 10 days.

(9.)

Let $x=$ the number cast for the first, and $y=$ the number cast for the second.

Then $x+y=375$,
and $x-y=91$;
which give $x=233$, and $y=142$

(10.)

Let $x=$ the value of the poorest horse and $y=$ that of the other:

Then, by 1st condition, $x+50=2y$,
and by 2d condition, $y+50=3x$,
which give $x=£30$, and $y=£40$

(11.)

In this example, we must bear in mind that the minute hand goes entirely round the face of the clock, while the hour hand passes from one hour to the other: that is, *the minute hand travels twelve times as fast as the hour hand.*

If, then, we suppose the face of the clock to be divided into twelve equal parts corresponding to the hours, and x and y to represent the distances passed over by the hour and minute hands, from the time of separating until they are again together, we shall have

$$12x=y,$$

and

$$y-x=12;$$

since, when the hands come together, the minute hand will have gained the entire twelve spaces on the hour hand. Multiplying the second equation by 12, and adding them together, we have

$$12y=y+144,$$

or

$$y=\frac{144}{11}=13\tfrac{1}{11}:$$

that is, the minute hand will have gone once around the face, and $1\frac{1}{11}$ of the hour spaces in addition; consequently the *time* required will be 1 hour, 5 minutes, $\frac{1}{11}$ of 5 minutes, or $\frac{5}{11}$ of one minute.

If we subtract the second from the first equation of condition, we have

$$11x=12, \text{ and } x=\frac{12}{11}=1\tfrac{1}{11};$$

that is, x is equal to 1 and $\frac{1}{11}$ of the hour spaces, which, reduced to *time*, gives 1 hour $5\frac{5}{11}$ minutes, as before.

(12.)

Denote by x the portion of the beer which the man would drink in a single day.

Then, by the conditions of the question, the man and woman together would drink $\frac{1}{12}$ of the cask in a single day, and the woman $\frac{1}{30}$ of it: hence, what the man would drink must be equal to the difference; that is,

$$x=\frac{1}{12}-\frac{1}{30}=\frac{30-12}{360}=\frac{18}{360}=\frac{1}{20}:$$

that is, the man will drink $\frac{1}{20}$ of the beer in a single day, and hence, the whole of it in 20 days.

(13.)

Let the fresh water to be added be denoted by x. Then the amount of the mixture will be denoted by $x+32$. But the addition of the fresh water will not increase the quantity of salt in the 32 lbs. of salt water; hence, the $x+32$ pounds of the mixture will contain one pound or 16 ounces of salt. But by the conditions of the question, 32 lbs. of this mixture are to contain 2 ounces of salt:

hence, $x+32 : 32 :: 16oz : 2oz.$;

consequently, $2x+64=512$;

or $x=224$.

(14.)

In this example, we must bear in mind that if the *rate* of interest be divided by 100, and the quotient multiplied by the principal, the product will always be the *amount* of interest.

Let $x=$ the greater part, and y the lesser.

Then $x+y=100000$;

also, $\frac{5x}{100}=$ what the larger part produced,

and $\frac{4y}{100}=$ interest of lesser part,

consequently, $\frac{5x}{100}+\frac{4y}{100}=4640$,

from which we find $x=64000$, and $y=36000$.

(15.)

Denote the number of votes received by the successful candidate by x, and the number received by the other, by y. Then, by the first condition, $x-y=1500$.

Had the first received $\frac{1}{4}$ of y in addition, the second would have received $y-\frac{1}{4}y=\frac{3}{4}y$, and we should have

$$x+\frac{1}{4}y=3\times\frac{3}{4}y-3500,$$

or $$4x+y=9y-14000;$$

from which we have $x=6500$, and $y=5000$.

(16.)

Let $x=$ the value of the gold watch, and y that of the silver watch.

Then, $$x+25=3\tfrac{1}{2}y=\frac{7}{2}y,$$

$$y+25=\frac{x}{2}+15.$$

that is, $$2x+50=7y,$$

and $$2y+50=x+30,$$

from which we have $x=80$, and $y=30$

(17.)

The separate figures which are placed by the side of each other, in order to express any number, are called *digits.* Now, from the relative value of these figures, resulting from the *places* which they occupy, we can easily see how the numbers may be expressed. For example, if the number is expressed by two digits, then the first figure on the right, plus ten times the second figure, will always give the number. Thus $36=3\times10+6$; and $87=8\times10+7$, &c.

If the number is expressed by three figures, then one hundred times the left-hand figure, plus ten times the middle figure, plus the right-hand figure, will express the number. Thus, $246=100\times2+10\times4+6=200+40+6=246$.

Let $x=$ the left-hand digit, and $y=$ the other.

Then. $$x+y=11;$$

also $$x+13=3y,$$

from which we have $x=5$ and $y=6$

(18.)

Let $x=$ the number of gentlemen, and $y=$ the number of ladies. Then $y-15=$ the ladies who remained, and $x-45$ $=$ the gentlemen who remained. And, by the conditions of the question,

$$x=2(y-15)=2y-30,$$

and $5(x-45)=y-15$, or $5x-225=y-15$,

from which we have $x=50$, and $y=40$.

(19.)

Let $x=$ the value of the horse, and $y=$ the number of tickets. If he sells the tickets at \$2, he will receive \$$2y$, if at \$3, he will receive \$$3y$.

Then $2y=x-30$,

and $3y=x+30$,

which give $x=150$, and $y=60$.

(20.)

Let $x=$ the amount of wheat purchased, and $y=$ the amount of rye.

Then $100x+75y=11750$ cents,

also $100\times\frac{1}{4}x+75\times\frac{1}{5}y=2750$ cents,

or $25x+15y=2750$ cents,

from which we have $x=80$, and $y=50$

[Page 116.]

(6.)

Let x, y, and z denote the separate ages of A, B, and C

Then $x=2y$, $y=3z$,

and $x+y+z=140$;

from which we find $x=84$, $y=42$, and $z=14$.

Or, this example may be solved with a single unknown quantity. Thus: let $x=$ C's age, then

$$\text{C's age} = x,$$
$$\text{B's age} = 3x,$$
$$\text{C's age} = 6x,$$

and $x+3x+6x=10x=140$,

whence $x=14$.

(7.)

Let $x=$ the cost of the horse; $y=$ the cost of the harness; and $z=$ the cost of the chaise.

Then, $x+y+z=\pounds 60$;

also $x=2y$, and $z=2(x+y)$;

from which we find the several answers.

But we may resolve the question by means of but a single unknown quantity Thus, let $x=$ the price of the harness.

Then, $x=$ the cost of the harness,

$2x=$ the price of the horse,

and $6x=$ the cost of the horse and harness;

Also, $x+2x+6x=9x=\pounds 60$, or $x=\pounds 6$ 13*s.* 4*d.*

[8.]

Let x, y, and z be the three parts.

Then, $x+y+z=36$,

also $\frac{1}{2}x=\frac{1}{3}y$, and $\frac{1}{3}y=\frac{1}{4}z$,

or $3x=2y$, and $4y=3z$;

from which we find $x=8$, $y=12$, and $z=16$.

This example may be resolved by only two unknown quantities. Thus, let x and y represent the first and second numbers, then will $36-x-y$ denote the third, and we shall have

$$\frac{1}{2}x=\frac{1}{3}y, \text{ or } 3x=2y,$$

and $$\frac{1}{3}y=\frac{36-x-y}{4}, \text{ or } 4y=108-3x-3y;$$

from which we find x and y, as before, equal to 8 and 12.

(9.)

Let x, y, and z represent, respectively, the *parts* of the work which A, B, and C would do in a single day, and let the whole work to be done be denoted by S. Then, in one day, A would do xS work, in two days, $2xS$ work, in three days, $3xS$ work, &c.; and the same for the others. Hence, by the conditions:

$$8xS+8yS=S$$
$$9xS+9zS=S$$
$$10yS+10yS=S$$

nd, by dividing by S, we have

$$8x+8y=1$$
$$9x+9z=1$$
$$10y+10z=1$$

from which equations we find $x=\frac{49}{720}$, $y=\frac{41}{720}$, and $z=\frac{31}{720}$, the parts of the work that each person will do in a single day.

Then, if each can do in one day the part of the work represented by each of these fractions, it is plain that the number of times which 1 contains each of the fractions, will express the number of days in which each person would do the whole work. That is:

A would do it in $\dfrac{1}{\frac{49}{720}}=\dfrac{720}{49}=14\dfrac{34}{49}$ days,

B in $\dfrac{1}{\frac{41}{720}}=\dfrac{720}{41}=17\dfrac{23}{41}$ days,

C in $\dfrac{1}{\frac{31}{720}}=\dfrac{720}{31}=23\dfrac{7}{31}$ days.

(10.)

Let x, y, and z denote the sums with which each began to play.

Then $x+y+z=600$,

first game, $x+\frac{1}{2}y=2y$;

second game, $x+\frac{1}{2}y-z=x$;

from which we find $x=\$300$, $y=\$200$, and $z=\$100$.

(11.)

Let x, y, and z denote the sums possessed by each.

Then $x+y+z=3640$,

second condition, $x+400=y-400+320$,

third condition, $y+140=z-140$;

from which we have $x=800$. $y=1280$, and $z=1560$.

(12.)

Let $x=$ the amount of the bill, $y=$ the amount possessed by A, and $z=$ the amount possessed by B; and let it be remembered that C has \$8.

Then, first condition, $y+\frac{1}{4}z=x$,

second condition, $z+1=x$,

third condition, $\frac{y}{2}+8=x$;

from which we have $x=\$13$, $y=10$, and $z=12$.

(13.)

We may again remark here, that if the rate of interest be divided by 100, and the quotient multiplied by the principal, the product will always be the amount of interest.

Let x denote the rate of interest received on the 1st sum. Then, $x+1$ and $x+2$, will be the other rates. Let y denote the capital of the first; then $y+10000$, and $y+15000$, will denote respectively the capital of the second and third.

Then, by 1st condition, $\frac{xy}{100}=\frac{(x+1)}{100}\times(y+10000)-800$,

and, by 2d condition, $\frac{xy}{100}=\frac{x+2}{100}\times(y+15000)-1500$

That is, $xy=xy+10000x+y-70000$,

and $xy=xy+15000x+2y-120000$;

or, $0=10000x+y-70000$,

and $0=15000x+2y-120000$;

from which we find $x=4$ and $y=30000$; and hence the other two sums are easily found.

(14.)

Let $x=$ a daughter's share,

then $2x=$ what each son received;

also, $3x+4x=$ what the children received,

$3x+4x+1000=$ widow's share

hence, $6x+8x+1000=15000$,

or $14x=15000-1000=14000$,

or $x=1000$.

(15.)

Let the sum to be divided be denoted by x.

Then, A's share $=\frac{x}{2}-3000$,

B's share $=\frac{x}{3}-1000$,

C's share $=\frac{x}{4}+800$.

Then the sum $x=\frac{x}{2}+\frac{x}{3}+\frac{x}{4}-3200,$

or $24x=12x+8x+6x-76800$

hence, $2x=76800$, or $x=\$38400$.

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(10.)

Let the number be denoted by x.

Then, $\frac{1}{3}x\times\frac{1}{4}x=108;$

that is, $\frac{1}{12}x^2=108$, or $x^2=1296$

hence, $x=\sqrt{1296}=36$.

(11.)

Let the number be denoted by x.

Then, $\frac{1}{5}x\times\frac{1}{6}x\div10=3;$

that is, $\frac{x^2}{30}\div10=3$, or $\frac{x^2}{300}=3;$

hence, $x^2=900$, and $x=\sqrt{900}=30$.

(12.)

Let the number be denoted by x.

Then, $x^2+18=\frac{x^2}{2}+30\frac{1}{2};$

hence, $2x^2+36=x^2+61$;
consequently, $x=\sqrt{25}=5$.

(15.)

Let $x=$ the greater: then $\frac{3}{4}x=$ the less.

Then $x^2-(\frac{3}{4}x)^2=28$,

that is, $x^2-\frac{9}{16}x^2=28$,

or $16x^2-9x^2=448$, and $7x^2=448$;

hence, $x^2=64$, and $x=8$.

(16.)

Let $x=$ the greater; then $\frac{5}{11}x=$ the less.

Then, $x^2+\frac{25}{121}x^2=584$,

and $121x^2+25x^2=70664$,

hence $146x^2=70664$, and $x^2=484$;

hence $x=22$.

(17.)

Denote the age of the elder by x; then $\frac{x}{4}=$ the age of the younger.

Then $x^2-\frac{1}{16}x^2=240$,

and $16x^2-x^2=3840$, or $x=16$

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(3.)

Let x and y represent the numbers.

Then, $xy=30$, and $\frac{x}{y}=3\frac{1}{3}=\frac{10}{3}$;

from which we have $x=10$, and $y=3$.

This question may be solved by a single unknown quantity Thus, if x represent one of the numbers, $\frac{30}{x}$ will represent the other,

and $$\frac{x}{\frac{30}{x}}=3\frac{1}{3}=\frac{10}{3};$$

that is, $$x^2=\frac{10\times 30}{3}=100, \text{ or } x=10.$$

(4.)

Let the numbers be represented by x and y.

Then, $$xy=a, \text{ and } \frac{x}{y}=b.$$

Then $x=by$, and, substituting this value in the first equation, we have

$$by^2=a, \text{ and } y=\sqrt{\frac{a}{b}}.$$

Then, by squaring the first equation, and substituting for y^2 its value, we have

$$x^2y^2=a^2, \text{ and } x^2\frac{a}{b}=a^2;$$

or $$x^2=ab, \text{ and } x=\sqrt{ab}.$$

(5.)

Let the two numbers be denoted by x and y.
Then, by first condition, $x^2+y^2=117$;
by second condition, $x^2-y^2=45$;
from which we have $x=9$, and $y=6$.

(6.)

Let the two numbers be denoted by x and y.
Then $x^2+y^2=a$,
and $x^2-y^2=b$;
hence, $x=\sqrt{\frac{a+b}{2}}$, and $y=\sqrt{\frac{a-b}{2}}$.

(7.)

Denote the numbers by x and y.
Then $x : y :: 3 : 4$, or $4x=3y$.
and $x^2+y^2=225$;
from which we have $x=9$, and $y=12$.

(8.)

Denote the numbers by x and y.
Then $x : y :: m : n$, or $nx=my$,
and $x^2+y^2=a^2$.

Squaring the first equation, and multiplying the second by m^2, we have, by transposing in the second,

$$n^2x^2=m^2y^2,$$

and

$$m^2x^2=m^2a^2-m^2y^2;$$

and then, by addition, we obtain

$$x^2(m^2+n^2)=m^2a^2,$$

and $$x=\frac{ma}{\sqrt{m^2+n^2}}; \text{ also } y=\frac{na}{\sqrt{m^2+n^2}}.$$

(9.)

Let the largest number be represented by $2x$; hen the ess will be denoted by x. We shall then have

$$4x^2-x^2=75,$$

and $$3x^2=75, \text{ and } x^2=25, \text{ or } x=5.$$

(10.)

Let the numbers be represented by x and y.

Then $$x : y :: m : n, \text{ or } nx=my,$$

and $$x^2-y^2=b^2;$$

from which we find $x=\frac{mb}{\sqrt{m^2-n^2}}$, and $y=\frac{nb}{\sqrt{m^2-n^2}}$.

(11.)

Let the amount placed at interest be represented by x.

Then $\frac{8}{100}\times x=$ the interest for one year,

and $\frac{4}{100}\times x=$ the interest for six months.

Then $$\frac{4x}{100}\times x=\frac{x^2}{25}=562500,$$

and $$x^2=14062500, \text{ and } x=3750.$$

(12.)

Let $x=$ the number of women, and $y=$ the number of boys.

Then $x : y :: 3 : 4$, or $4x=3y$.

and $x=\frac{3}{4}y$,

and $x+y=$ the number of persons,

$\frac{x+y}{2}=$ what the boys receive,

and $2y=$ what the women receive.

Then $\frac{x+y}{2}+2y=138$,

and $x+y+4y=276$.

Then $\frac{3}{4}y+y+4y=276$,

and $3y+4y+16y=1104$;

or $23y=1104$, and $y=48$.

Note. This, it will be seen, is an equation of the *first degree*, and is placed among those of the second degree, to lead the student to have confidence in his own method, and not to rely too implicitly on the arrangements of the author.

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(6.)

Let $x=$ the number of sheep which he purchased.

Then $\frac{60}{x}=$ the cost of a single sheep,

or $\frac{1200}{x}=$ the cost of a single sheep in shillings,

and $\frac{1200\times 15}{x}=\frac{18000}{x}=$ *cost* of fifteen sheep.

Then the number of sheep sold will be represented by $x-15$, and $2(x-15)=$ the amount of profit. Now, had there been no profit, the amount received for the sheep would have been just equal to the cost, less the value of the fifteen unsold; and consequently, the amount received, less the profit, must be just equal to this difference. That is, reducing to shillings,

$$1080-2(x-15)=1200-\frac{18000}{x};$$

hence $1080x-2x^2+30x=1200x-18000$;

and, by reducing and dividing by the co-efficient of x^2, we have

$$x^2+45x=9000;$$

from which, by taking the positive value, we have $x=75$.

(7.)

Let the number of pieces be denoted by x: and by reducing the cost and the amount received to shillings, we find that he paid 675 shillings, and sold for 48 shillings per piece.

Then, $\frac{675}{x}=$ the price per piece in shillings,

and $48x=$ what he received for the whole.

But what he received, minus what he gave, must be equal to his profits; that is, to the cost of a single piece. That is,

$$48x-675=\frac{675}{x}$$

and $$48x^2-675x=675,$$

or $$x^2-\frac{675}{48}x=\frac{675}{48},$$

and by completing the square,

$$x^2-\frac{675}{48}x+\frac{(675)^2}{(96)^2}=\frac{675}{48}+\frac{(675)^2}{(96)^2}$$
$$=\frac{1350}{96}+\frac{(675)^2}{(96)^2}$$
$$=\frac{1350+96}{(96)^2}+\frac{(675)^2}{96}$$
$$=\frac{585225}{(96)^2}.$$

Then, by extracting the square root of both members, and taking the positive root, which answers to the question in its arithmetical sense, we have

$$x-\frac{675}{96}=\frac{765}{96},$$

and $$x=\frac{675}{96}+\frac{765}{96}=\frac{1440}{96}=15.$$

(8.)

Let x represent the digit which stands in the ten's place, and y the digit which stands in the unit's place

Then, $10x+y=$ the number.

By the first condition, $\frac{10x+y}{xy}=3,$

and by the second, $10x+y+18=10y+x;$

from which we readily find $x=2$, and $y=4$.

(9.)

Let the number be denoted by x.

Then, $(10-x)x=21;$

hence, $10x-x^2=21,$

or, $x^2-10x=-21,$

completing the square, $x^2-10x+25=-21+25=4;$

hence, $x=5\pm\sqrt{4}=5\pm2=7$ or 3

(10.)

Let the distance travelled by B be denoted by x; then the distance travelled by A will be represented by $x+18$.

Now, the rate of travel, or the distance travelled in a single day, will be found by dividing the distance by the number of days; hence,

$\frac{x}{15\frac{3}{4}}=$ what A would travel in one day,

and $\frac{x+18}{28}=$ what B would travel in one day.

Now, the entire distance travelled by each, divided by the distance which each travelled in one day, will give the *time* in which B travelled x miles, and in which A travelled $x+18$ miles. But since the time was the same,

$$\frac{x+18}{\left(\frac{x}{15\frac{3}{4}}\right)}=\frac{x}{\left(\frac{x+18}{28}\right)}$$

and by reducing,

$$(x+18)\times(x+18)\times15\tfrac{3}{4}=x\times x\times28,$$

or $$(x+18)\times(x+18)\times\frac{63}{4}=x\times x\times28;$$

that is, $$63x^2+2268x+20412=112x^2,$$

or $$49x^2-2268x=20412,$$

and $$x^2-\frac{2268}{49}x=\frac{20412}{49};$$

completing the square, $$x^2-\frac{2268}{49}x+\frac{(2268)^2}{(98)^2}=\frac{20412}{49}+\frac{(2268)^2}{(98)^2}$$

and $$x^2-\frac{2268}{49}x+\frac{(2268)^2}{(98)^2}=\frac{40824\times98}{(98)^2}+\frac{(2268)^2}{(98)^2}$$

which, after performing the operations indicated, gives $x=54$

(11.)

Denote the less number by x. Then the greater will be denoted by $x+15$; and we shall have

$$\frac{(x+15)x}{2}=x^3,$$

and, dividing by x, $x+15=2x^2$,

and, by transposing, $x^2-\frac{1}{2}x=7.5$,

and, completing the square, $x^2-.5x+.0625=7.5625$;

hence, $x=.25\pm\sqrt{7.5625}=3$,

by taking the positive root.

(12.)

Let the greater number be denoted by x, and the less by y

Then, by first condition, $(x+y)x=77$ (1.);

by second condition, $(x-y)y=12$ (2.);

that is, $x^2+xy=77$ (3.),

and, $xy-y^2=12$ (4.).

By adding, we have $x^2-y^2+2xy=89$,

and, by transposing, $x^2-y^2=89-2xy$.

If we multiply the first and second equations together, we obtain

$$(x^2-y^2)xy=924,$$

and hence $x^2-y^2=\frac{924}{xy}$.

Placing this value of x^2-y^2 equal to that found above, and we have

$$89-2xy=\frac{924}{xy},$$

or $89xy-2x^2y^2=924$;

and, placing $xy=z$, we obtain

$$89z-2z^2=924;$$

and hence, by changing the signs and dividing, we have,

$$z^2-44.5z=-462.$$

Then, by completing the square,

$$z^2-44.5z+495.0625=33.0625;$$

hence, $z=22.25\pm\sqrt{33.0625}$,

or $z=22.25\pm5.75$,

or $z=28$, or $z=16.5$.

Substituting the first value of z for xy, in equations (3.) and (4.), gives $x=7$ and $y=4$; and substituting the second value $z=16.5$, for xy in the same equations, we find

$$x=\frac{11}{2}\sqrt{2}, \quad \text{and} \quad y=\frac{3}{2}\sqrt{2}.$$

(13.)

Let the numbers be denoted by x^2 and y^2.

Then, $x^2+y^2=100$, (1.)

and $x+y=14$. (2.)

From the second equation we have, by transposing,

$x=14-y$, and by squaring,

$$x^2=196-28y+y^2.$$

Substituting this value in equation (1.), we have

$$196-28y+y^2+y^2=100;$$

and, by reducing, $y^2-14y=-48$.

Completing the square, we have

$$y^2-14y+49=-48+49=1,$$

and $y=+7\pm1=8$;

or, if we take the minus sign, then $y=6$. If we take $y=8$, we find $x=6$, and if we take $y=6$, we find $x=8$; hence the numbers are 64 and 36.

(14.)

Let the numbers be denoted by x and y.

Then, $x+y=24$,

and $xy=35(x-y)=35x-35y$.

From the first equation, we have

$$x=24-y.$$

Substituting this value in the second, we obtain

$$y(24-y)=35(24-y)-35y,$$

that is, $24y-y^2=840-35y-35y$;

hence, $y^2-94y=-840$.

Completing the square, $y^2-94y+2209=1369$,

and $y=47+37=84$, or 10.

If we take the first root, 84, the value of x will be -60, and these two numbers will satisfy the two *equations* of condition. But the *enunciation* of the question required the number 24 to be *divided* into two parts, and this required that neither x nor y should have a value exceeding 24; hence, we must take the second value of $y=10$. This gives $x=14$.

(15.)

Let the numbers be denoted by x and y.

Then, $x+y=8$, (1.)

and $x^3+y^3=152$. (2.)

By cubing both numbers of equation (1.), we have

$$x^3+3x^2y+3xy^2+y^3=512. \quad (3);$$

and, by subtracting the second equation from the third, we

have $3x^2y+3xy^2=360$; and, dividing by 3,
we obtain $x^2y+xy^2=120$,
or $xy(x+y)=120$; but, since in equation (1.)
$x+y=8$, we have

$$8xy=120, \text{ or } xy=15.$$

Combining this with equation (1.) we readily find $x=3$, and $y=5$.

(16.)

Let the number of yards sold by the first, be denoted by x, and the number sold by the second by y.

Now, if the whole amount received, *for any number of things sold*, be divided by the number of things, the quotient will be the *cost of each thing*. Hence, if 24 dollars be divided by the number of yards of stuff sold by the second, the quotient will be the amount per yard received by the first; and for a like reason, $12\frac{1}{2}$ divided by x will be the amount per yard received by the second.

That is, $\frac{24}{y}=$ what the first received per yard,

and $\frac{12\frac{1}{2}}{x}=$ what the second received per yard.

But, the first sold x yards, and the second y yards· and, if the amount per yard be multiplied by the number of yards the product will be the amount received. Hence,

$$\frac{24}{y}\times x+\frac{12\frac{1}{2}}{x}\times y=35;$$

and, by the second condition, $y-x=3$, or $y=x+3$

Then, by clearing the first equation of fractions, we have

$$24x^2+12\tfrac{1}{2}y^2=35xy;$$

and, by substituting for y its value, $x+3$, we obtain,

$$24x^2+12\tfrac{1}{2}(x^2+6x+9)=35x(x+3);$$

that is, $24x^2+12\tfrac{1}{2}x^2+75x+112\tfrac{1}{2}=35x^2+105x$,

and reducing, $1\tfrac{1}{2}x^2-30x=-112\tfrac{1}{2}$,

and, dividing by $1\tfrac{1}{2}$, we have

$$x^2-20x=-75;$$

which gives, $x=10\pm5=15$, or 5;

from which we have the corresponding values of $y=18$ or $y=8$.

(17.)

Let the highest rate of interest be denoted by y, and the smallest by z. Now, as the incomes are to be equal, it is plain that the first sum put at interest will be the least, which let us denote by x. Then the larger or second part will be denoted by $13000-x$. Then, since the amount of interest on any sum is equal to the sum multiplied by the rate divided by 100, we have,

by first condition, $x\times\frac{y}{100}=(13000-x)\times\frac{z}{100}$,

by second condition, $x\times\frac{z}{100}=360$,

by third condition, $(13000-x)\frac{y}{100}=490$

Clearing the fractions, we have

$$xy=13000z-zx, \quad (1)$$

$$xz=36000, \quad (2.)$$

and $$13000y-xy=49000. \quad (3.)$$

If, now, we substitute the value of xz from equation (2.) in equation (1.), we shall have

$$xy=13000z-36000, \quad (4.);$$

then, adding together equations (3.) and (4.), we have

$$13000y=13000z+13000,$$

or, $$13y=13z+13,$$

or, $$y=z+1 \quad (5.).$$

Now, to eliminate x from equations (2.) and (3.), multiply the first by y, and the second by z, and we have

$$xyz=36000y,$$

and $$13000yz-xyz=49000z,$$

and, by adding, $$13000yz=36000y+49000z,$$

or, $$13yz=36y+49z.$$

Now, substituting for y its value in equation (5.), we have

$$13z(z+1)=36(z+1)+49z;$$

that is, $$13z^2+13z=36z+36+49z,$$

and $$z^2-\frac{72}{13}z=\frac{36}{13};$$

by completing the square, we have

$$3^2-\frac{72}{13}z+\frac{(36)^2}{(13)^2}=\frac{36}{13}+\left(\frac{36}{13}\right)^2$$

$$=\frac{36\times13}{(13)^2}+\left(\frac{36}{13}\right)^2$$

Hence, $$z=\frac{36}{13}\pm\frac{42}{13}=\frac{78}{13}=6.$$

The negative value of z is not applicable to the question.

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(2.)

$$l=a-(n-1)r.$$

Make $a=90$, $r=4$, and $n=15$;

then, $l=90-(15-1)4=90-56=34$.

(3.)

$$l=a-(n-1)r.$$

Make $a=100$, $n=40$, and $r=2$;

then $l=100-(40-1)2=100-78=22$

(4.)

$$l=a-(n-1)r.$$

Make $a=80$, $n=10$, and $r=4$;

then $l=80-(10-1)4=80-36=44$.

(5.)

$$l=a-(n-1)r$$

Make $a=600$, $n=100$, and $r=5$;

then $l=600-(100-1)5=600-495=105$.

(6.)

$$l=a-(n-1)r.$$

Make $a=800$, $n=200$, and $r=2$;

then $l=800-(200-1)2=800-398=402$.

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(2.)

$$S=\left(\frac{a+l}{2}\right)\times n.$$

Make $a=3$, $l=27$, and $n=12$;

then $S=\frac{3+27}{2}\times 12=180.$

(3.)

$$S=\left(\frac{a+l}{2}\right)\times n.$$

Make $a=4$, $l=20$, and $n=10$;

then $S=\left(\frac{4+20}{2}\right)\times 10=120.$

(4.)

$$S=\left(\frac{a+b}{2}\right)\times n.$$

Make $a=100$, $b=200$, and $n=80$;

then $S=\left(\frac{100+200}{2}\right)\times 80=12000.$

(5.)

$$S=\left(\frac{a+b}{2}\right)\times n.$$

Make $a=500$, $b=60$, and $n=20$;

then $S=\left(\frac{500+60}{2}\right)\times 20=5600$

[251]

(6.)

$$S=\left(\frac{a+b}{2}\right)n.$$

Make $a=800$, $b=1200$, and $n=50$;

then $S=\left(\frac{800+1200}{2}\right)\times 50=50000$

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(2.)

$$r=\frac{l-a}{n-1}.$$

Make $l=22$, $a=4$, and $n=10$;

then $r=\frac{22-4}{10-1}=\frac{18}{9}=2.$

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(2.)

$$l=a+(n-1)r.$$

Make $a=2$, $n=100$, and $r=7$;

then $l=2+(100-1)7=2+693=695.$

(3.)

First, to find the last term. We have

$$l=a+(n-1)r.$$

and, making $a=1$, $n=100$, and $r=2$, we have

$$l=1+(100-1)2=1+198=199;$$

then $S=\left(\frac{a+b}{2}\right)\times n=\frac{1+199}{2}\times 100=10000$

(4.)

To find the least term. We have

$$l=a-(n-1)r;$$

and, making $a=70$, $n=21$, and $r=3$, we have

$$l=70-(21-1)\times3=70-60=10.$$

Then, $$S=\left(\frac{a+l}{2}\right)\times n;$$

and, making $a=70$, $l=10$, and $n=21$, we have

$$S=\left(\frac{70+10}{2}\right)\times21=840$$

(5.)

To find the last term, we have

$$l=a+(n-1)r.$$

and, making $a=4$, $n=8$, and $r=8$, we have

$$l=4+(8-1)8=4+56=60.$$

Then $$S=\left(\frac{a+l}{2}\right)\times n;$$

and, making $a=4$, $l=60$, and $n=8$, we have

$$S=\left(\frac{4+60}{2}\right)\times8=256.$$

(6.)

$$r=\frac{b-a}{n-1}.$$

Make $b=20$, $a=2$, and $n=10$, and we have

$$r=\frac{20-2}{10-1}=\frac{18}{9}=2.$$

(7.)

$$\frac{b-a}{n+1}.$$

Make $b=19$, $a=4$, and $n=4$; then we have

$$r=\frac{19-4}{5}=\frac{15}{5}=3\,;$$

hence, 4 . 7 . 10 . 13 . 16 . 19 form the series.

(8.)

First, to find the last term, we have

$$l=a-(n-1)r$$

Make $a=10$, $n=21$, and $r=\frac{1}{3}$; then

$$l=10-(21-1)\tfrac{1}{3}=10-6\tfrac{2}{3}=3\tfrac{1}{3}\,;$$

then $S=\left(\frac{10+3\frac{1}{3}}{2}\right)\times 21=\frac{\frac{30}{3}+\frac{10}{3}}{2}\times 21=\frac{40}{6}\times 21=\frac{840}{6}=140$

(9.)

We have the equations,

$$S=\left(\frac{a+l}{2}\right)\times n, \quad \text{and} \quad l=a+(n-1)r.$$

In these equations all the quantities are known, except a and l Substituting the numbers for the known quantities, we have

$$2945=\left(\frac{a+185}{2}\right)\times n, \quad \text{and} \quad 185=a+(n-1)6$$

From the second equation we have

$$a=191-6n.$$

Substituting this value of a in the first equation, after having cleared the fraction, we obtain

$$5890=(191-6n+185)n,$$

that is, $$5890=191n-6n^2+185n;$$

hence, $$n^2-\frac{376}{6}n=-\frac{5890}{6}.$$

Completing the square, we have

$$n^2-\frac{376}{6}n+\frac{(188)^2}{6^2}=-\frac{5890}{6}+\frac{(188)^2}{6^2}.$$

$$=-\frac{35340}{6^2}+\frac{35344}{6^2}=\frac{4}{6^2}.$$

Then, $$n=+\frac{188}{6}\pm\sqrt{\frac{4}{6^2}}=\frac{188}{6}\pm\frac{2}{6}$$

that is, $$n=\frac{190}{6}, \text{ or } n=\frac{186}{6}=31.$$

Now, as the number of terms in any series must be expressed by a whole number, we know that n cannot be *fractional:* hence, we must use the negative root as applicable to the question, and, consequently, $n=31$.

Then, $$l=a+(n-1)r,$$

gives $$a=185-(n-1)r=185-180=5.$$

(10.)

We have from Art. **142**,

$$r=\frac{b-a}{m+1}.$$

Making $b=5$, and $a=2$, and $m=9$, we have

$$r=\frac{5-2}{10}=0.3;$$

from which the terms are easily found.

(11.)

We have the formula,

$$S=\left(\frac{a+l}{2}\right)\times n.$$

Now, $a=1$, and $l=n$; hence

$$S=\left(\frac{1+n}{2}\right)\times n=n\left(\frac{n+1}{2}\right).$$

(12.)

The formula, for the last term,

$$l=a+(n-1)\times r;$$

making $a=1$, and $r=2$, we have

$$l=1+(n-1)2=1+2n-2=2n-1.$$

Then, in the formula,

$$S=\left(\frac{a+l}{2}\right)n,$$

substitute for l its value, and for a its value 1, and we have

$$S=\left(\frac{1+2n-1}{2}\right)\times n=n\times n=n$$

(13.)

In this example we know that the person must travel four yards to place the first stone in the basket, and that he must travel four yards in addition for each successive stone which he brings. Hence, we have the first term, the common difference, and the number of terms, to find the sum of the series.

First, to find the last term, we have

$$l=a+(n-1)r.$$

Making $a=4$, $n=100$, and $r=4$, we have

$$l=4+(100-1)4=4+396=400.$$

Then, $$S=\left(\frac{a+l}{2}\right)\times n=\left(\frac{4+400}{2}\right)\times 100=20200 \text{ yards},$$

which, divided by 1760, the number of yards in a mile, gives 11 miles and 840 yards.

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(6.)

Here we have given the first term, the common ratio, and the number of terms, to find the last term.

Hence, $l=1\times 2^9=1\times 512=512$ cents.

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(4.)

In this example we have the first term, the common ratio, and the number of terms given, to find the last term and the sum of the series:

$$l=1\times 2^{11}=1\times 2048=2048.$$

Then, to find the sum of the series, we have

$$S=\frac{lq-a}{q-1},$$

in which $l=2048$, $q=2$, and $a=1$; hence

$$S=\frac{4096-1}{1}=4095.$$

(5.)

In this example we have given the first term, the common ratio, and the number of terms, to find the sum of the series

First, to find the last term, we have

$$l=1\times2^{11}=2048.$$

Then, $S=\dfrac{lq-a}{q-1}=\dfrac{4096-1}{1}=4095$ shillings,

which is equal to £204, 15*s*.

(6.)

In this example we have the first term, the ratio, and the number of terms, to find the last term and the sum of the series. We have

$$l=1\times3^{9}=1\times19683=19683 \text{ cents.}$$

$$S=\frac{lq-a}{9-1}=\frac{19683\times3-1}{2}=29524 \text{ cents.}$$

(7.)

In this example we have the first term, the ratio, and th number of terms, to find the last term and the sum of the series.

$$l=4\times 8^{15}=4\times 35184372088832=140737488355328.$$

For the sum of the series, we have

$$S=\frac{140737488355328\times 8-4}{8-1},$$

that is, $S=160842843834660.$

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(3.)

First, to find the last term, we have

$$l=aq^5,$$

and making $a=512$, and $q=\frac{1}{4}$, we have

$$l=512\times(\tfrac{1}{4})^5=512\times\frac{1}{1024}=\tfrac{1}{2}$$

Then, $$S=\frac{a-lq}{1-q}=\frac{512-\frac{1}{8}}{\frac{3}{4}}=682\tfrac{1}{2}.$$

(4.)

First, to find the last term, we have

$$l=aq^6=2187\times(\tfrac{1}{3})^6=3.$$

Then, $$S=\frac{a-lq}{1-q}=\frac{2187-1}{\frac{2}{3}}=3279$$

(5.)

First, to find the last term, we have

$$=aq^5=972\times(\tfrac{1}{3})^5=4.$$

Then, $$S=\frac{a-lq}{1-q}=\frac{972-\frac{4}{3}}{\frac{2}{3}}=1456.$$

(6.)

$$l=aq^7=147456\times(\tfrac{1}{4})^7=9.$$

Then, $$S=\frac{a-lq}{1-q}=\frac{147456-\frac{9}{4}}{\frac{3}{4}}=196605.$$

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(2.)

$$\sqrt{16\times4}=\sqrt{64}=8.$$

(3.)

$$\sqrt{27\times3}=\sqrt{81}=9.$$

(4.)

$$\sqrt{72\times2}=\sqrt{144}=12.$$

$$\sqrt{64\times4}=\sqrt{256}=16.$$

ANSWERS TO QUESTIONS IN ADDITION AND SUBTRACTION.

(1.)

$x^n(a+b+c+d)$.

(2.)

$x^n(a+b-c-d)$

(3.)

$13a^4$

(4.)

$9a^3+a^4$

(5.)

$16a^2b^m-3a^m$

(6.)

$11a^3b^3-5ab^2c-11a^mb^5-9a^2b^3-2b^x$

(7.)

$8a^4b-3a^2b^3c$

(8.)

$a^mb^p(6-3c)+a^3b^{m-1}(2g^2+3)+7a^3+3a^2b^2$.

(9.)

$a^3b^2c^4(9-h)-2a^nb^m+11b+4c^x-8d^5$.

(10.)

$-7b^mcx^2-7$

(11.)

$2a^4+8a^3b^2+4cd^2+7d+3a^2$.

(12.)

$14ab^m-2d^5+11a^4b^n$

(13.)

$-32a^3b^6-11a^mb^n+12ac^5$.

(14.)

$16a^3b^3c^5$.

(15.)

$-9ab+12a^2b^2-4a$.

ANSWERS TO QUESTIONS IN MULTIPLICATION.

(1.)

a^{m+n}

(2.)

$-42a^{13}$.

(3.)

$-60a^{15}bd$.

(4.)

$15a^{2p+8q+5}fcx$.

(5.) $-150a^{12}b^9c.$

(6.) $392a^{10}b^{16}c^3d.$

(7.) $a^{2m+n+1}b^{p+r+1}c^{q+1}q.$

(8.) $4a^4b-12a^3b^2-20a^2b^3$

(9.)

$6a^5b^6c^2-15a^4bc^3+27a^5b^3c^5.$

(10.)

$-56h^9l^6-16h^4l^8+24ah^7l^7-56h^4l^5.$

(11.)

$-2a^3b^5c^2d+2b^6c^3d^4f-6bc^{m+2}d.$

(12.)

$a^{4m+1}b^{3m+2}-6a^{2m-1}b^{p+m-1}+a^{3m+3}b^{2m-1}.$

(13.)

$3k^4-26k^3l+37k^2l^2-14kl^3.$

(14.)

$6f^7+7f^6l-65f^5l^2+12f^4l^3.$

(15.)

$20a^5-88a^4x+47a^3x^2-6a^2x^3.$

(16.) $a^8-a^2.$

(17.) $a^5+32b^5.$

(18.) $4a^8x^4-9b^8y^4.$

(19.)

$21a^7-43a^6b+150a^5b^2-110a^4b^3-104a^3b^4-32a^2b^5.$

(20.)

$7a^{10}-25a^8b^2+48a^6b^4-23a^4b^6+5a^2b^8.$

ANSWERS TO QUESTIONS IN DIVISION.

(1.) a^{m-n}

(2.) a^{m-2n}

(3.) $4a^{12}$

(4.) $\frac{c}{d}a^{12}$.

(5.) $2(a+b)^3$

(6.) $(a+x)\times(a+y)$

(7.) $2ab^2—5f+9a^2bx$

(8.) c^3

(9.) a^2+ab+b^2

(10.) $3a^3—5a^2b+2ab^2$

(11.) $a^4—4a^3b^3+6a^2b^6$.

(12.) $a^2—b^2$

(13.) $a^2b^3—5a^5b^4—2a^3b^5$.

(14.) $a^6+2a^4z^2+4a^2z^4+8z^6$.

(15.) $a^2—5ab+6b^2$.

(16.) $2c^2+3bc—b^2$.

(17.) $1+an+a^2n^2$.

(18.) $a^4+a^3z+az^3+z^4$

(19.) $1—6z+9z^2$.

(20.) $a^4+4a^3x+12a^2x^2+16ax^3+16x^4$.

(21.) $ad—cd$.

ANSWERS TO QUESTIONS IN REDUCTION OF FRACTIONS.

(1.) $\frac{6c}{d}—\frac{2bc}{a}—\frac{2}{3f}$.

(2.) $—4a+3b—\frac{2c}{a}—\frac{1}{2a}$

(3.)

$$\frac{3c}{ab^2}-\frac{f}{ab^2}+\frac{3gh}{4a^2b^2}.$$

(4.)

$a.$

(5.)

$$\frac{ax-a^2-bx+ab+cx}{x-a}.$$

(6.)

$$\frac{ax^2-cx-b+y}{ax-c}.$$

(7.)

$$\frac{ad+bd+y-a}{d}$$

(8.)

$$\frac{ax-a^2b-x^2+abx-c+d}{a-x}.$$

(9.)

$$\frac{ax-ay-b-c+d}{x-y}.$$

(10.)

$$\frac{6abf^2x+9abf-b-ax}{b}.$$

(11.)

$$\frac{5a^2c^2x-facy-x+ax}{fac}.$$

(12.)

$$3ab^2-5f+\frac{7a^2bx.}{2}$$

(13.)

$$\frac{9x^2}{8}-\frac{1}{3a}+\frac{9b}{2ax^2}.$$

(14.)

$a^2-ax+b.$

(15.)

$$\frac{a^2f-2afx+fx^2}{fa^2-fx^2},\quad \frac{a^2b-bx^2-a^2c+cx^2}{fa^2-fx^2},$$

$$\frac{4a^2fx-acf+4afx^2-cfx}{fa^2-fx^2}.$$

(16.)

$$\frac{3a^2b-3abx}{9a^2bx-3ab^2-9abx^2+3b^2x},\quad \frac{3a^3x-6a^2x^2-a^2b-bx^2+2abx+3ax}{9a^2bx-3ab^2-9abx^2+3b^2x}$$

$$\frac{-9abcx+3b^2c}{9a^2bx-3ab^2-9abx^2+3b^2x}.$$

(17.)

$$\frac{4acfy-cxy}{7acy-c^2y},\qquad -\frac{7a^2y-7axy-acy+cxy}{7acy-c^2y},$$

$$\frac{35a^2c^2-5ac^3}{7acy-c^2y}$$

(18.)

$$\frac{16a^2x^2+4abx}{32a^2cx-4afx},\qquad \frac{64a^2c^2-16acf+f^2}{32acx-4afx},$$

(19.)

$$-\frac{a^3+2a^2x+ax^2}{a^3-ax^2},\qquad \frac{a^3-a^2b-a^2x+abx}{a^3-ax^2},$$

$$\frac{a^2c-cx^2-a^2d+dx^2}{a^3-ax^2}.$$

(20.)

$$\frac{acx+bcx-c^2x+a^2c+abc-ac^2}{cx^2-a^2c},\qquad \frac{cx^2-a^2c+fx^2-a^2f}{cx^2-a^2c},$$

$$\frac{cx^2-2acx+ca^2}{cx^2-a^2c}.$$

ANSWERS IN ADDITION, SUBTRACTION, MULTIPLICATION AND DIVISION.

(1.)

$$\frac{a^2-ax+bx-ab+2acx-a^2c-cx^2}{2ax-a^2-x^2}.$$

(2.)

$$\frac{a^2-ax-ac+cx+bc-ab+aby-bcy}{ab-bc}$$

(3.)

$$\frac{9acd-3adf+3bcx-3abcy-bfx+abfy+4abdx}{3bcd-bdf}.$$

(4.)

$$\frac{-ax^2+fx+ac^2-cg+a^2c-cg-5acxy-5a^2xy}{-ac-ax}.$$

(5.)

$$\frac{7a^2x-5a^3+7abx-5a^2b+x^2-ax}{ax+bx-a^2-ab}.$$

(6.)

$$\frac{8a^2b+3a^2-8abx-3ax-6ab+bx}{ab-bx}$$

(7.)

$$\frac{8cx-acx-3bc+5abx}{bc}.$$

(8.)

$$\frac{4a^2xy+3a^2y^2-3a^2cx^2+a^2cfx-a^2fy}{a^2cxy-a^2y^2}.$$

(9.)

$$\frac{9a^2xy-9ax-9d-27a^2xy-6a^2x^2+axy}{9ax}.$$

(10.)

$$\frac{40a^2y-13axy-448a^2b^2x+48ax^2+56ab^2x}{64ax-8x^2}$$

(11.)

$$\frac{-42ax-3bx+42a^2+3ab}{6x+6a}.$$

(12.)

$$\frac{15a^2xy-25a^2x+5ax^2}{x^2-ax}.$$

(13.)

$$\frac{162a^3+36a^2x-15ax^2+x^3}{27a^2+9ax}.$$

(14.)

$$\frac{-15abxy+25aby-5bx-3a^2xy+5a^2y-ax}{-by}.$$

(15.)

$$\frac{-36a^2b+30a-48a^2+6ab^2-5b+8ab}{b^3-a^2b}.$$

(16.)

$$\frac{3c}{2}-dc-\frac{f}{2a}+\frac{c}{2ad}$$

(17.)

$$\frac{a}{3c}+\frac{fd^2}{6c^2}-ad+\frac{7d}{3c}$$

(18.)

$$a+\frac{b}{2}-2c$$

(19.)

$$3f-\frac{7h}{2}+\frac{x}{3}$$

(20.)

$$\frac{bx}{6}+\frac{3y}{4}+5z.$$

ANSWERS TO EXAMPLES IN EQUATIONS OF THE FIRST DEGREE.

(1.)

$y=8,\ x=6$

(2.)

$x=5,\ y=3$

(3.)

$$x=\frac{4cg-4ag+3cf-3af}{4ac-4a^2+3c-3a+4}$$

(4.)

$$x=\frac{20cd+20a-4bc-5ac}{20-9c}.$$

(5.)

$x=9,\ y=8$

(6.)

$x=10,\ y=12$

(7.)

$x=6,\ y=5$

(8.)

$$x=\frac{9b}{12}$$

(9.)

$$x=\frac{bcf-3ac+2ab+bcd}{3b+bc-6c}.$$

(10.)

$$x=\frac{c+ab-bd}{a-b},\quad y=\frac{a(ab-bd)+bc}{b(a-b)}.$$

(11.)

$x=17$

(12.)

$x=4,\ y=6,\ z=8.$

(13.)

$x=1,\ y=2,\ z=3.$

(14.)

$x=10,\ y=12,\ z=14.$

(15.)

$x=-12,\ y=50.$

(16.)

$x=3,\ y=5.$

(17.)

$$x=\frac{bc}{a+b},\quad y=\frac{ac}{a+b}$$

(18.)

$$x=\frac{cg-bh}{ag-bf},\quad y=\frac{ah-cf}{ag-bf}.$$

(19.)

$$x=\frac{2b^2-6a^2+d}{3a},\ y=\frac{3a^2-b^2+d}{3b}.$$

(20.)

$$x=\frac{a}{bc},\ y=\frac{a+2b}{c}.$$

(21.)

$$x=\frac{bf}{b-f},\quad y=\frac{bf}{b+f}.$$

(22.)

$x=16,\ y=7\frac{3}{4},\ z=5\frac{1}{2}.$

(**23**.)

$$x=17,\ \ y=22,\ \ z=45.$$

(**24**.)

In this example we must not proceed to clear the equations of fractions, but if we subtract the second from the first, and then add the third we shall find y; after which we shall find

$$x=\frac{2}{a+b-c},\ \ y=\frac{2}{a-b+c},\ \ z=\frac{2}{b+c-a},$$

(**25**.)

$$x=\frac{ce-bf}{ae-bd},\ \ y=\frac{af-cd}{ae-bd},\ \ z=\frac{a(el-fg)-d(bl-cg)}{h(ae-bd)}$$

ANSWERS TO QUESTIONS IN EQUATION OF THE SECOND DEGREE

(**1**.)

$x=8$ and $x=-2\frac{1}{4}$.

(**2**.)

$x=5$ and $x=-4\frac{1}{5}$.

(**3**.)

$x=22\frac{4}{5}$ and $x=18\frac{2}{3}$.

(**4**.)

$x=-2\frac{1}{7}$ and $x=5\frac{1}{2}$.

(**5**.)

$x=6\frac{3}{7}$ and $x=3\frac{1}{4}$

(**6**.)

$x=60\frac{2}{3}$ and $x=16\frac{2}{7}$.

(**7**.)

$x=6\frac{3}{7}$ and $x=3\frac{1}{4}$.

(**8**.)

$x=-25\frac{10}{39}$ and $x=-52$

(**9**.)

$x=4+\sqrt{30}$ and $x=4-\sqrt{30}$.

(**10**.)

$$x=\frac{-1+\sqrt{85}}{6},\ x=\frac{-1-\sqrt{85}}{6},$$

(11.)

$$x=\frac{118+\sqrt{13724}}{5} \text{ and } x=\frac{118-\sqrt{13724}}{5}.$$

(12.)

$$x=1+\sqrt{-9}, \text{ and } x=1-\sqrt{-9}$$

(13.)

$$x=\frac{7+\sqrt{-1039}}{16} \text{ and } x=\frac{7-\sqrt{-1039}}{16}.$$

(14.)

$x=15\frac{1}{2}$ and $x=-16\frac{1}{2}$.

(15.)

$x=-46$, $x=24\frac{1}{3}$.

(16.)

$x=67\frac{1}{6}$ and $x=4\frac{1}{2}$.

(17.)

$x=14\frac{3}{5}$ and $x=\frac{72}{245}$

(18.)

$x=7\frac{22}{113}$ and $x=2\frac{1}{2}$.

(19.)

$x=6\frac{1}{2}$ and $x=\frac{1}{2}$.

(20.)

$x=15\frac{1}{2}$ and $x=-16\frac{1}{2}$.

(21.)

$x=14$ and $x=-10$

(22.)

$x=9$ and $x=1\frac{2}{13}$

(23.)

$x=10$ and $x=-\frac{2}{3}$.

(24.)

$x=5\frac{2}{3}$ and $x=5$.

PROMISCUOUS QUESTIONS

IN EQUATIONS OF THE FIRST DEGREE.

1. A person expends 30 cents for apples and pears, giving one cent for four apples, and one cent for five pears: he then sold, at the prices he gave, half his apples and one-third his pears, for 13 cents. How many did he buy of each? First, let $x=$ the number of apples, and $y=$the pears;

then, $\frac{x}{4}=$ the amount he paid for the apples,

and $\frac{y}{5}=$ the amount he paid for the pears:

also, $\frac{x}{4}+\frac{y}{5}=30$, what he paid for both.

By the condition of the sale, $\frac{1}{2}x$, and $\frac{1}{3}y$, at the same rate, must have brought 13 cents:

$\frac{1}{2}x\times\frac{1}{4}=\frac{1}{8}x=$ what he got for the apples; and

$\frac{1}{3}y\times\frac{1}{5}=\frac{1}{15}y=$ what he got for the pears

Also, $\frac{1}{8}x+\frac{1}{15}y=13$ cents: therefore,

$$\frac{x}{4}+\frac{y}{5}=30, \quad \text{and} \quad \frac{1}{8}x+\frac{y}{15}=13,$$

are the equations of condition; from which we find $x=72$, and $y=60$

2. A tailor cut 19 yards from each of three equal pieces of cloth, and 17 yards from another of the same length, and found that the four remnants were together equal to 142 yards. How many yards in each piece?

Let the length of each piece be denoted by x. Then

$3x-57=$ what remained of the first three pieces,

and $x-17=$ what remained of the fourth piece.

Hence, $4x-74=142$ yards, what remained in all.

Therefore, $4x=142+74$, or $x=54$.

3. A fortress is garrisoned by 2600 men, consisting of infantry, artillery, and cavalry. Now, there are nine times as many infantry, and three times as many artillery soldiers as there are cavalry. How many are there of each corps?

Let the number of cavalry soldiers be denoted by x.

Then, $3x=$ the artillery, and $9x=$ the infantry: also,

$$x+3x+9x=2600, \text{ or } x=200.$$

4. All the journeyings of an individual amounted to 2970 miles. Of these he travelled $3\frac{1}{2}$ times more by water than on horseback, and $2\frac{1}{4}$ times more on foot than by water. How many miles did he travel in each way?

Let $x=$ the number of miles he travelled on horseback

Then, $3\frac{1}{2}x=$ what he travelled by water, and

$$3\tfrac{1}{2}x\times2\tfrac{1}{4}=\frac{63}{8}x= \text{ what he travelled on foot.}$$

Consequently, $x+3\frac{1}{2}x+\dfrac{63}{8}x=2970$;

rom which we find $x=240$; hence, he travelled by water 340 miles, and on horseback 1890 miles.

5. A sum of money was divided between two persons, A and B. A's share was to exceed B's in the proportion of 5 to 3, and to exceed $\frac{5}{9}$ of the entire sum by 50. What was the share of each?

Let B's share be denoted by x. Then, $\frac{5}{3}x=$ A's, and

$$x+\frac{5}{3}x=\frac{8x}{3}=\text{ the entire sum.}$$

But, by the condition, $\frac{5}{3}x-\frac{5}{9}$ of $\frac{8}{3}x=\frac{5}{3}x-\frac{40}{27}x=50$;

that is, $\frac{45}{27}x-\frac{40}{27}x=50$, or $\frac{5}{27}x=50$, or $x=270$.

Hence, A's share is 450.

6. There are 52 pieces of money in each of two bags, out of which A and B help themselves. A takes twice as much as B left, and B takes seven times as much as A left. How much did each take?

Let $x=$ what A took, and $y=$ what B took.

Then, $52-x=$ what A left, and $52-y=$ what B left

But, by the conditions,

$x=2(52-y)$, and $y=7(52-x)$,

that is, $x=104-27$, and $y=364-7x$.

Hence, $x=48$, and $y=28$.

7. Two persons, A and B, agree to purchase a house together, worth \$1200. Says A to B, give me two-thirds of

your money and I can purchase it alone; but, says B to A, if you give me three-fourths of your money I shall be able to purchase it alone. How much had each?

Let x denote what A had, and y what B had.

Then, $x+\frac{2}{3}y=1200$, and $y+\frac{3}{4}x=1200$,

from which we have $x=$ \$800, and $y=$ \$600.

8. To divide the number a into three such parts, that the second may be m times, and the third n times greater than the first.

Let the first be denoted by x, then the second will be denoted by mx, and the third by nx.

Hence, $x+mx+nx=a$, which gives,

$$\text{1st, } x=\frac{a}{1+m+n},\ \text{2d, } \frac{ma}{1+m+n},\ \text{3d, } \frac{na}{1+m+n}.$$

8. A father directs that \$1170 shall be divided among his three sons, in proportion to their ages. The oldest is twice as old as the youngest, and the second is one-third older than the youngest. How much was each to receive?

Let $x=$ the portion of the youngest.

Then, $x+\frac{1}{3}x=$ the portion of the second,

and $2x=$ the portion of the third.

By the condition, $x+x+\frac{1}{3}x+2x=$ \$1170,

that is, $3x+3x+x+6x=3510$, and $x=270$.

9. Three regiments are to furnish 594 men, and each to furnish in proportion to its strength. Now, the strength of

the first is to the second as 3 to 5; and that of the second to the third as 8 to 7? How many must each furnish?

Let x denote the complement of the first.

Then, $\frac{5}{3}x=$ that of the second,

and $\frac{7}{8}$ of $\frac{5}{3}x=\frac{35}{24}x=$ that of the third,

and $x+\frac{5}{3}x+\frac{35}{24}x=594$;

that is, $72x+120x+105x=42768$, and $x=144$.

10. A grocer finds that if he mixes sherry and brandy in the proportion of 2 to 1, the mixture will be worth 78*s.* per dozen; but if he mixes them in the proportion of 7 to 2, he can get 79*s.* a dozen. What is the price of each liquor per dozen?

Let $x=$ the price of the brandy, and $y=$ that of the sherry.

If, now, we make the first mixture, that is, two dozen of sherry and one dozen of brandy, the mixture itself will contain three dozens, and will, consequently, be worth $78s.\times3=234$. Hence, we have

$$2x+y=234, \text{ for the first, and}$$

$$7x+2y=79\times9=711; \text{ for the second;}$$

which equations give $x=81$, and $y=72$.

11. A person bought 7 books, the prices of which were in arithmetical progression, (in shillings.) The price of the one next above the cheapest, was 8 shillings, and the price of the dearest, 23 shillings. What was the price of each book?

Excluding the first book, we have the two extremes and number of terms given to find the common difference. We find the formula on page 225 of the Elementary Algebra,

$$r=\frac{b-a}{m+1}, \text{ which gives } r=\frac{23-8}{5}=3;$$

hence, the cost of the books is

5, 8, 11, 14, 17, 20, 23, shillings respectively.

12. A number consists of three digits, which are in arithmetical proportion. If the number be divided by the sum of the digits, the quotient will be 26; but, if 198 be added to it, the digits will be inverted.

Let the digits be denoted by x, y, and z.

Then, $x : y :: y : z$, and $x+z=2y$;

also, $100x+10y+z$ will express the number, and

$$\frac{100x+10y+z}{x+y+z}=26;$$

that is, $100x+10y+z=26x+26y+26z$.

3d condition, $100x+10y+z+198=10z+10y+x$.

From these equations, we find the values to be, 2, 3, and 4, and, consequently, the number to be 234.

13. A person has three horses, and a saddle which is worth $220. If the saddle be put on the back of the first horse, it will make his value equal to that of the second and third; if it be put on the back of the second, it will make his value double that of the first and third; if it be put on the back of the third, it will make his value triple that of the first and second. What is the value of each horse?

Let their values be denoted, respectively, by x, y, and z

Then, $x+220=y+z$,

2d condition, $y+220=2(x+z)=2x+2z$,

3d condition, $z+220=3(x+y)=3x+3y$;

from which we find $x=20$, $y=100$, and $z=140$.

14. The crew of a ship consisted of her complement of sailors, and a number of soldiers. There are 22 sailors to every three guns, and 10 over; also, the whole number of hands is five times the number of soldiers and guns together. But after an engagement, in which the slain were one-fourth of the survivors, there wanted 5 men to make 13 men to every two guns. Required, the number of guns, soldiers, and sailors.

Let the number of guns be denoted by x, the number of soldiers by y, and the sailors by z. Now, as there are 22 seamen to every three guns, there will be $\frac{22}{3}$ seamen to each gun, and

$$\frac{22}{3}\times x, \text{ for } x \text{ guns; hence,}$$

1st condition gives $z=\frac{22}{3}\times x+10$,

or $3z=22x+30$. (1.)

2d condition, $y+z=5(y+x)=5y+5x$. (2.)

Now, if we denote the number of slain by s,

hen $y+z-s=$ the survivors, and

$$\frac{1}{4}(y+z-s)=s, \text{ or } s=\frac{1}{5}(y+z).$$

Then, $y+z-\frac{1}{5}(y+z)=\frac{4}{5}(y+z)=$ survivors, and by

3d condition, $\frac{4}{5}(y+z)+5=\frac{13}{2}x;$

that is, $8y+8z+50=65x;$ (3.)

from which three equations, we find $x=90$, $y=55$, and $z=670$.

15. Three persons have $96, which they wish to divide equally between them. In order to do this, A, who has the most, gives to B and C as much as they have already: then B divides with A and C in the same manner, that is, by giving to each as much as he had after A had divided with them: C then makes a like division with A and B, when it is found that they all have equal sums. How much had each at first?

Let x, y, and z denote the sums which they respectively had at first.

Then, $x-y-z=$ what A had,

$2y=$ what B had,

and $2z=$ what C had,

after the division with A. Also,

$2x-2y-2z=$ what A had,

$2y-(x-y-z)-2z=$ what B had,

and $4z=$ what C had,

after the division with B. Again,

$4x-4y-4z=$ what A had,

$4y-2(x-y-z)-4z=$ what B had

$4z-(2x-2y-2z)-[2y-(x-y-z)-2z]=$ what C had, after his division with A and B.

But these three last sums are all equal to each other, and the sum of x, y, and z is equal to 96. Hence, after reducing, we have

$$x+y+z=96,$$
$$6x-10y=2z,$$
and
$$5x-3y=11z;$$

from which we find $x=\$52$, $y=\$28$, and $z=\$16$.

16. To divide the number a into three such parts, that the first shall be to the second as m to n, and the second to the third as p to q.

Let the parts be denoted by x, y, and z.

Then, $x+y+z=a$,

and $x : y :: m : n$, or $nx=my$;

also, $y : z :: p : q$, or $qy=pz$;

From these equations we find,

$$x=\frac{mpa}{mp+np+nq},\ y=\frac{npa}{mp+np+nq},\ z=\frac{nqa}{mp+np+nq}.$$

17. Five heirs, A, B, C, D, and E, are to divide an inheritance of \$5600. B is to receive twice as much as A, and \$200 more; C three times as much as A, less \$400; D the half of what B and C receive together, and \$150 more; and E the fourth part of what the four others get, plus \$475. How much did each receive?

Let $x=$ A's portion.

Then, $2x+200=$ B's,

$$3x-400=\text{C's},$$
$$2\tfrac{1}{2}x+50=\text{D's},$$
$$\frac{8\frac{1}{2}x-150}{4}+475=\text{E's};$$

and $8\frac{1}{2}x-150+\dfrac{8\frac{1}{2}x-150}{4}+475=5600$, the estate,

or, $34x-600+\dfrac{17}{2}x-150+1900=22400$,

or, $68x-1200+17x-300+3800=44800$;

hence $85x=42500$, and $x=500$.

18. A person has four casks, the second of which being filled from the first, leaves the first four-sevenths full. The third being filled from the second, leaves it one-fourth full, and when the third is emptied into the fourth, it is found to fill only nine-sixteenths of it. But the first will fill the third and fourth, and leave 15 quarts remaining. How many quarts does each hold?

Let $x=$ the number of quarts that will fill the first cask.

Then, $x-\dfrac{4}{7}x=\dfrac{3}{7}x=$ what fills the second,

$\dfrac{3}{7}x-\dfrac{1}{4}\left(\dfrac{3}{7}x\right)=\dfrac{9}{28}x=$ content of third cask,

and $\dfrac{9}{28}x=\dfrac{9}{16}$ the content of fourth:

or, conten of fourth, $=\dfrac{16}{28}x=\dfrac{4}{7}x.$

Then. $x=\dfrac{9}{28}x+\dfrac{4}{7}x+15,$

or, $$28x=9x+16x+420,$$

and $$x=140.$$

19. A courier who had started from a place 10 days, was pursued by a second courier. The first travels 4 miles a day, the other 9. How many days before the second will overtake the first?

Let the number of days be denoted by x.

Then, since the first courier travels four miles a day, we have

$$4x+10\times4=4x+40=\text{ the distance}$$

travelled by the first courier; and

$$9x=\text{ the distance travelled by the second};$$

hence, $$9x=4x+40, \quad \text{or} \quad x=8.$$

20. If the first courier had left n days before the other, and made a miles a day, and the second courier had travelled b miles, how many days before the second would have overtaken the first?

Now, let $x=$ the number of days. Then

$$ax+na=bx,$$

and hence, $$x=\frac{na}{b-a}.$$

21. A courier goes $31\frac{1}{2}$ miles every five hours, and is followed by another after he had been gone eight hours. The second travels $22\frac{1}{2}$ miles every three hours. How many hours before he will overtake the first?

The rate of travel of the first is $31\frac{1}{2}\div5=6.3$ **miles**; and of the second $22\frac{1}{2}\div3=7\frac{1}{2}=7.5$

If we substitute these numbers for n, a, and b, in the last formula, we have

$$x=\frac{8\times 6.3}{7.5-6.3}=\frac{50.4}{1.2}=42 \text{ hours after the departure of the 2d.}$$

22. Two places are eighty miles apart, and a person leaves one of them and travels towards the other, at the rate of $3\frac{1}{2}$ miles per hour. Eight hours after, a person departs from the second place, and travels at the rate of $5\frac{1}{6}$ miles per hour How long before they will meet each other?

The first will have travelled $3\frac{1}{2}\times 8=28$ miles, at the time the second departs; hence, they will be 52 miles apart. Now, if we denote by x the number of hours after the departure of the second, until they meet, we shall have

$3\frac{1}{2}x=$ distance travelled by the first, after the second starts, and $5\frac{1}{6}x=$ the distance travelled by the second:

hence, $$3\tfrac{1}{2}x+5\tfrac{1}{6}x=52,$$

or $$\frac{7}{2}x+\frac{31}{6}x=52;$$

$$42x+62x=624, \quad \text{or} \quad x=6 \text{ hours.}$$

23. Three masons, A, B, and C, are to build a wall. A and B together can do it in 12 days; B and C in 20 days; and A and C in 15 days. In what time can each do it alone, and in what time can they all do it if they work together?

Instead of denoting the *parts* of the work done, by A, B, and C, by x, y, and z, as in Example 9, page 24, let us denote the times in which each would perform the work, respectively, by x, y, and z. and denote the work to be done by 1. Now, if it takes A, x days to do the work he will, in one day, do a part of the work denoted by $\frac{1}{x}$; hence.

$$\frac{1}{x} = \text{the part A can do in a day,}$$

$$\frac{1}{y} = \text{what B could do,}$$

$$\frac{1}{z} = \text{what C could do,}$$

and these, multiplied by any number of days, would give what each could do in those days. Hence,

1st condition, $\frac{12}{x}+\frac{12}{y}=1$, or $\frac{1}{x}+\frac{1}{y}=\frac{1}{12}$, (1.)

2d, $\frac{20}{y}+\frac{20}{z}=1$, or $\frac{1}{y}+\frac{1}{z}=\frac{1}{20}$, (2.)

3d, $\frac{15}{x}+\frac{15}{z}=1$, or $\frac{1}{x}+\frac{1}{z}=\frac{1}{15}$; (3.)

Subtracting the second equation from the first, we have

$$\frac{1}{x}-\frac{1}{z}=\frac{1}{12}-\frac{1}{20}=\frac{8}{240}=\frac{1}{30}.$$

Then, adding the third to this, we have

$$\frac{2}{x}=\frac{1}{30}+\frac{1}{15}=\frac{3}{30}, \quad \text{or} \quad x=\frac{60}{3}=20.$$

and $y=30$, and $z=60$.

Now, the three together could do in one day

$$\frac{1}{20}+\frac{1}{30}+\frac{1}{60}=\frac{3}{60}+\frac{2}{60}+\frac{1}{60}=\frac{6}{60}=\frac{1}{10}$$

of the work; hence, they could do the whole work in ten days

24. A laborer can do a certain work expressed by a, in a time expressed by b; a second laborer, the work c in a time d; a third, the work e in a time f. It is required to find the time it would take the three laborers, working together, to perform the work g

If the first does a work in b days, the work done in a single day will be denoted by $\frac{a}{b}$; the work done in a single day by the second, by $\frac{c}{d}$; and by the third, by $\frac{e}{f}$. If, now, we denote by x the time in which the three would be employed in doing the work denoted by g, we see that what each would do in x days, will be expressed by what he would do in one day, multiplied by x; and, since the work done by them all in x days is equal to g, we have

$$\frac{a}{b}\times x+\frac{c}{d}\times x+\frac{e}{f}\times x=g,$$

and clearing the fractions, we obtain

$$adfx+bcfx+bdex=bdfg,$$

and,
$$x=\frac{bdfg}{adf+bcf+bde}.$$

If we make

$$a=27,\ b=4,\ c=35,\ d=6,\ e=40,\ f=12,\ g=191,$$

x will be found to equal 12.

25. Required to find three numbers with the following conditions. If 6 be added to the 1st and 2d, the sums are to one another as 2 to 3. If 5 be added to the 1st and 3d the

sums are as 7 to 11; but, if 36 be subtracted from the 2d and 3d, the remainders will be as 6 to 7.

Let the numbers be denoted by x, y, and z. Then,

1st condition, $x+6 \ : \ y+6 \ :: \ 2 \ : \ 3$, which gives

$$3x+18=2y+12$$

2d condition, $x+5 \ : \ z+5 \ :: \ 7 \ : \ 11$, which gives

$$11x+55=7z+35.$$

3d condition, $y-36 \ : \ z-36 \ :: \ 6 \ : \ 7$, which gives

$$7y-252=6z-216;$$

from which we find, $x=30$, $y=48$, $z=50$.

26. The sum of $500 was put out at interest, in two separate sums, the smaller sum at two per cent. more than the other. The interest of the larger sum was afterwards increased, and that of the smaller diminished, by one per cent. By this, the interest of the whole was augmented one-fourth. But if the interest of the greater sum had been so increased, without any diminution of the less, the interest of the whole would have been increased one-third. What were the sums, and what the rate per cent.?

Let the larger sum be denoted by x. Then will the smaller be represented by $500-x$. Denote the higher rate of interest by y; then will the lower rate be represented by $y-2$. The interest received on the larger sum will be expressed by

$$x\times\frac{y-2}{100},$$

and that received on the smaller, by

$$(500-x)\times\frac{y}{100},$$

and the amount received on the two sums, by

$$x\times\frac{y-2}{100}+(500-x)\times\frac{y}{100}.$$

Now, after the rates of interest are changed, they will be represented by $y-1$, and $y-1$, and the interest on the whole amount will be expressed by

$$x\times\frac{y-1}{100}+(500-x)\times\frac{y-1}{100}.$$

But this interest, by the conditions of the question, exceeds, by one-fourth, that received under the first supposition: that is, it is equal to *five-fourths* of that interest. Hence,

$$\frac{5}{4}\left[x\times\frac{y-2}{100}+(500-x)\times\frac{y}{100}\right]=x\times\frac{y-1}{100}+(500-x)\times\frac{y-1}{100}$$

But, under the supposition that the interest on the smaller sum had not been changed, the new interest accruing would have been *one-third* greater than the first interest—that is equal to *four-thirds* of that interest. Hence,

$$\frac{4}{3}\left(x\times\frac{y-2}{100}+(500-x)\times\frac{y}{100}\right)=x\times\frac{y-1}{100}+(500-x)\times\frac{y}{100}$$

From the first equation of condition, we find, after reducing,

$$5(xy-2x+500y-xy)=4(xy-x+500y-xy-500+x),$$

or, $$-10x+2500y=2000y-2000;$$

that is, $$50y-x=-200. \quad (1.)$$

And, from the second equation of condition, we have

$$4(xy-2x+500y-xy)=3(xy-x+500y-xy);$$

that is, $2000y-8x=1500y-3x.$

Hence, $500y=5x,$ or $x=100y.$ (2)

Substituting this value in equation (1.), we find $y=4$, and $x=400$.

27. The ingredients of a loaf of bread weighing 15*lbs.*, are rice, flour, and water. The weight of the rice, augmented by 5*lbs.*, is two-thirds the weight of the flour; and the weight of the water is one-fifth the weight of the flour and rice together. Required, the weight of each.

Let $x=$ the weight of the rice, and y that of the flour

Then, $x+5=\frac{2}{3}y,$

and $\frac{1}{5}(x+y)=$ weight of the water.

Then, $x+y+\frac{1}{5}(x+y)=15,$

from which equations, $x=2$, and $y=10\frac{1}{2}$.

28. Several detachments of artillery divided a certain number of cannon balls. The first took 72 and $\frac{1}{9}$ of the remainder; the next 144 and $\frac{1}{9}$ of the remainder; the third 216 and $\frac{1}{9}$ of the remainder; the fourth 288 and $\frac{1}{9}$ of what was left; and so on, until nothing remained; when it was found that the balls were equally divided. Required, the number of balls and the number of detachments.

Let the number of balls be represented by x.

Then, $72+\frac{1}{9}(x-72)=$ what the first took and $x-72-\frac{1}{9}(x-72)=$ what was left.

Also, $144+\frac{1}{9}[x-72-\frac{1}{9}(x-72)-144]=$ what the 2d took

But, by the conditions of the question, these sums are equal to each other. Hence,

$$144+\frac{1}{9}[x-72-\frac{1}{9}(x-72)-144]=72+\frac{1}{9}(x-72),$$

$$648+x-72-\frac{1}{9}(x-72)-144=x-72;$$

that is, $\frac{1}{9}(x-72)=504,$

and, $x=4608.$

Substituting this value in the expression for what the first detachment took, and we find

$$72+\frac{1}{9}(4608-72)=72+504=576.$$

Then, $4608\div 576=8$, the number of detachments.

29. A banker has two kinds of money; it takes *a* pieces of the first to make a crown, and *b* of the second to make the same sum. He is offered a crown for *c* pieces. How many of each kind must he give?

Let x and y, respectively, denote the number which he must take of each sort.

Then, since it takes *a* pieces of the first to make a crown, it follows that the *value* of one piece is equal to one crown divided by *a*.

That is, the value of one piece $=\frac{1\ crown}{a}$,

and for the second sort, we have

value of one piece $=\frac{1\ crown}{b}$.

Then, the value of x pieces of the first, and y pieces of the second, is equal to one crown. Hence,

$$x\times\frac{1\ crown}{a}+y\times\frac{1\ crown}{b}=1\ crown;$$

and by dividing by 1 crown, and reducing, we have

$$bx+ay=ab;$$

but, as the number of pieces taken was equal to c, we have

$$x+y=c;$$

from which two equations, we find

$$x=\frac{a(c-b)}{a-b}; \quad \text{and} \quad y=\frac{b(a-c)}{a-b}.$$

30. Find what each of three persons, A, B, and C is worth, knowing, 1st, that what A is worth, added to l times what B and C are worth, is equal to p; 2d, that what B is worth, added to m times what A and C are worth, is equal to q; 3d, that what C is worth, added to n times what A and B are worth, is equal to r.

If we denote what A, B, and C are respectively worth, by y, z, and s, we shall have

$$y+l(z+s)=p,$$
$$z+m(y+s)=q,$$
$$s+n(y+z)=r;$$

from which we can easily find the values of y, z, and s.

But we can resolve the question in another way, by denoting by x what A, B, and C are worth. We shall then have,

1st condition, $y+l(x-y)=p$,

2d condition, $z+m(x-z)=q$,

3d condition, $s+n(x-s)=r$;

from which we have

$$y=\frac{p-lx}{1-l},$$

$$z=\frac{q-mx}{1-m},$$

$$s=\frac{r-nx}{1-n};$$

and adding the equations, and substituting for $y+z+s$, their value x, we obtain

$$x=\frac{p-lx}{1-l}+\frac{q-mx}{1-m}+\frac{r-nx}{1-n},$$

and by reducing,

$$x(1-l)(1-m)(1-n)=(p-lx)(1-m)(1-n)+$$
$$(q-mx)(1-l)(1-n)+(r-nx)(1-l)(1-m);$$

and, by separating the multipliers of x in the second member, we have

$$x(1-l)(1-m)(1-n)=p(1-m)(1-n)-lx(1-m)(1-n)+$$
$$q(1-l)(1-n)-mx(1-l)(1-n)+r(1-l)(1-m)$$
$$-nx(1-l)(1-m),$$ and hence,

$$x=\frac{p(1-m)(1-n)+q(1-l)(1-n)+r(1-l)(1-m)}{(1-l)(1-m)(1-n)+l(1-m)(1-n)+m(1-l)(1-n)+n(1-l)(1-m)}.$$

31. Find the values of the estates of six persons, A, B, C, D, E, and F, from the following conditions. 1st. The sum of the estates of A and B is equal to a; that of C and D to b; and that of E and F to c. 2d. The estate of A is worth m times that of C; the estate of D is worth n times that of E, and the estate of F is worth p times that of B.

Let the estate of C be denoted by x. Then, by the conditions of the question, we have

$$\begin{array}{ll} \text{A's estate} & =mx, \\ \text{B's} & =a-mx, \\ \text{C's} & =x, \\ \text{D's} & =b-x, \\ \text{E's} & =\dfrac{b-x}{n}, \\ \text{F's} & =p(a-mx); \end{array}$$

by adding, and observing that the sum of the estates is equal to $a+b+c$, we have

$$mx+a-mx+x+b-x+\frac{b-x}{n}+p(a-mx)=a+b+c,$$

and by cancelling, and clearing the fraction,

$$na+nb+b-x+pna-pnmx=na+nb+nc,$$

and,
$$x=-\frac{nc-b-pna}{pnm+1};$$

from which the values of the remaining estates are readily found

PROMISCUOUS QUESTIONS

INVOLVING EQUATIONS OF THE SECOND DEGREE.

1. Find three numbers, such, that the difference between the third and second shall exceed the difference between the second and first by 6: that the sum of the numbers shall be 33, and the sum of their squares 467.

Let the second number be denoted by x, and the difference between the second and first by y.

Then, $x-y=$ 1st number,

$x=$ 2d number,

and $x+y+6=$ 3d number.

Then, $3x+6=33$, and hence, $x=9$;

also, $(x-y)^2+x^2+(x+y+6)^2=467$;

that is, $3x^2+12x+12y+2y^2=431$;

or, substituting for x its value 9,

$$351+12y+2y^2=431,$$

hence, $y^2+6y=40$,

and, $y=4$ or -10.

2. It is required to find three numbers in geometrical progression, such that their sum shall be 14, and the sum of their squares 84.

Let x and y denote the two extremes;

then, $\sqrt{xy}=$ mean number,

and by the conditions,

$$x+\sqrt{xy}+y=14,$$

and $$x^2+xy+y^2=84.$$

Dividing the second equation by the first, gives

$$x-\sqrt{xy}+y=6,$$

and by adding this to the first equation, and then subtracting it, we have

$$x+y=10, \quad \text{and} \quad \sqrt{xy}=4,$$

from which we find $x=2$, and $y=8$; and hence the numbers are 2, 4, and 8.

3. What two numbers are those, whose sum multiplied by the greater, gives 144, and whose difference multiplied by the less, gives 14?

Let the greater be denoted by x, and the less by y.

Then, $$(x+y)x=144, \quad (1.)$$

$$(x-y)y=14; \quad (2.)$$

and multiplying the equations together, we obtain,

$$(x^2-y^2)xy=2016. \quad (3.)$$

But equations (1.) and (2.) may be put under the form,

$$x^2+xy=144,$$

and $$xy-y^2=14, \quad \text{or} \quad xy=14+y^2,$$

and subtracting, $x^2+y^2=130$, or $x^2=130-y^2$.

Substituting in equation (3.), the value of $xy=144-x^2$, and then for x^2 its value $130-y^2$, and we obtain

$$(130-2y^2)(14+y^2)=2016;$$

that is, $1820-28y^2+130y^2-2y^4=2016$

Hence, $y^4-51y^2=-98.$

Then, placing $z=y^2$, and z^2 for y^4, we have

$$z^2-51z=-98,$$

which gives, by taking the positive root, which is the or corresponding to the arithmetical enunciation,

$$z=49, \quad \text{and, consequently,} \quad y^2=49, \quad \text{or} \quad y=7$$

The value of x is easily found equal to 9.

3. What number is that which, being divided by the product of its two digits, the quotient will be 3; and if 18 be added to it, the resulting number will be expressed by the digits inverted?

Let the left digit be denoted by x, and the right digit by y.

Then, $10x+y=$ the number,

and, $10y+x=$ the number expressed by the digits inverted.

Then, 1st condition, $\dfrac{10x+y}{xy}=3,$

and, 2d condition, $10x+y+18=10y+x;$

that is, $9x=9y-18.$

From the first equation we have

$$10x+y=3xy,$$

$$x=\frac{-y}{10-3x}.$$

Substituting this value of x in the second equation, and we have, $\dfrac{-9y}{10-3y}=9y-18;$

that is, $-9y=90y-180-27y^2+54y.$

Hence, $$27y^2-153y=-180,$$
and dividing by 9, we obtain
$$3y^2-17y=-20,$$
and this equation gives $y=4$.

4. What two numbers are those, which are to each other as m to n, and the sum of whose squares is b?

Let the numbers be denoted by x and y. Then,
$$x : y :: m : n, \quad \text{or} \quad nx=my,$$
and
$$x^2+y^2=b,$$
which give
$$x=\frac{m\sqrt{b}}{\sqrt{(m^2+n^2)}}, \quad y=\frac{n\sqrt{b}}{\sqrt{(m^2+n^2)}}.$$

5. What two numbers are those, which are to each other as m to n, and the difference of whose squares is b?

$$\text{Ans. } \frac{m\sqrt{b}}{\sqrt{(m^2-n^2)}}, \quad \frac{n\sqrt{b}}{\sqrt{(m^2-n^2)}}.$$

6. A certain capital is out at 4 per cent. interest. If we multiply the number of dollars in the capital by the number of dollars in the interest, at five months, we obtain \$$117041\frac{2}{3}$. What is the capital?

Let the capital be denoted by x. Then,
$$x\times\frac{4}{100}=\frac{4x}{100}= \text{ the interest for one year;}$$
and
$$\frac{4x}{100}\times\frac{5}{12}=\frac{20}{1200}x=\frac{1}{60}x= \text{ the interest for 5}$$
months. Then, $$\frac{1}{60}x\times x=117041\tfrac{2}{3};$$
or, $$x^2=7022500, \quad \text{and} \quad x=2650$$

7. A person has three kinds of goods, which together cost \$$230\frac{5}{24}$. One pound of each article costs as many times $\frac{1}{24}$ of a dollar as there are pounds of that article. Now, he has one-third more of the second kind than of the first, and $3\frac{1}{2}$ times more of the third than of the second. How many pounds had he of each?

Let $x=$ the number of pounds of the first.

Then, $$x+\frac{1}{3}x=\frac{4}{3}x= \text{ second,}$$

and, $$\frac{4}{3}x\times 3\tfrac{1}{2}=\frac{4}{3}x\times\frac{7}{2}=\frac{14}{3}x= \text{ third.}$$

Then, if one pound cost $\frac{1}{24}$ of x, x pounds will cost

$$x\times\frac{1}{24}x, \quad\text{or}\quad \frac{1}{24}x^2. \quad\text{Hence,}$$

$$x\times\frac{1}{24}x=\frac{1}{24}x^2= \text{ what the first cost.}$$

$$\frac{4}{3}x\times\frac{1}{24} \text{ of } \frac{4}{3}x=\frac{2}{27}x^2= \text{ what the second cost.}$$

$$\frac{14}{3}x\times\frac{1}{24} \text{ of } \frac{14}{3}x=\frac{49}{54}x^2= \text{ what the third cost.}$$

Then, $$\frac{1}{24}x^2+\frac{2}{27}x^2+\frac{49}{54}x^2= \$230\tfrac{5}{24}.$$

Now, as 216 is the least common divisor, we have,

$$9x^2+16x^2+196x^2=49725,$$

and $$x^2=225, \quad\text{or}\quad x=15;$$

from which we readily find the other numbers to be 20 and 70.

8. Required to find three numbers, such, that the product of the first and second shall be equal to a; the product of the first and third equal to b; and the sum of the squares of the second and third equal to c.

Let the numbers be denoted by x, y, and z.

Then, $xy=a$, $xz=b$, $y^2+z^2=c$.

From the first equation, we have

$$x=\frac{a}{y}, \text{ and hence, } \frac{az}{y}=b,$$

or

$$az=by, \quad\text{and}\quad y^2=\frac{a^2}{b^2}z^2;$$

hence,

$$\frac{a^2}{b^2}z^2+z^2=c, \quad\text{or}\quad z=b\sqrt{\frac{c}{a^2+b^2}},$$

$$y=a\sqrt{\frac{c}{a^2+b^2}}, \qquad x=\sqrt{\frac{c}{a^2+b^2}}.$$

9. It is required to find three numbers, whose sum shall be 38, the sum of their squares 634, and the difference between the second and first greater by 7 than the difference between the third and second.

Let the numbers be denoted by x, y, and z.

Then, $x+y+z=38$, (1.)

$x^2+y^2+z^2=634$, (2.)

and, $y-x=z-y+7$. (3.)

Adding the first and third equations, we have

$$3y=45, \quad\text{or}\quad y=15;$$

from which we easily find $x=3$, and $z=20$

10. Find three numbers in geometrical progression, whose sum shall be 52, and the sum of the extremes to the mean, as 10 to 3.

Let the first extreme be denoted by x, and the common ratio by r. Then, the mean number will be denoted by rx, and the other extreme by r^2x. Then,

by 1st condition, $x+rx+r^2x=52$,

by 2d condition, $x+r^2x : rx :: 10 : 3$;

or $3(x+r^2x)=10rx$.

Dividing by x, $3+3r^2=10r$,

which gives $r=3$.

This being substituted in the first equation, gives $x=4$; hence, the numbers are 4, 12, and 36.

11. The sum of three numbers in geometrical progression is 13, and the product of the mean by the sum of the extremes is 30. What are the numbers?

Let the numbers be denoted, respectively, by x, y, and z. Then, by the first condition,

$$x+y+z=13, \quad \text{or} \quad x+z=13-y, \quad (1.)$$

by 2d condition, $y(x+z)=30$, (2.)

and, $y^2=xz$. (3.)

If, now, we substitute in equation (2.), the value of $x+z$, taken from equation (1.), we have

$$y(13-y)=30, \quad \text{or} \quad 13y-y^2=30,$$

which gives $y=3$, and from which we readily find $x=1$, and $z=9$.

12. It is required to find three numbers, such, that the product of the first and second, added to the sum of their squares, shall be 37; and the product of the first and third, added to the sum of their squares, shall be 49; and the product of the second and third, added to the sum of their squares, shall be 61.

Let the numbers be denoted by x, y, and z.

Then, $xy+x^2+y^2=37$, (1.)

$xz+x^2+z^2=49$, (2.)

$yz+y^2+z^2=61$. (3.)

Subtracting the first equation from the second, we have

$$(z-y)x+z^2-y^2=12,$$

that is, $(z-y)x+(z+y)(z-y)=12$;

or, $(x+y+z)(z-y)=12$,

or, $$x+y+z=\frac{12}{z-y}.$$

Again, if we subtract the second equation from the 3d, we have, $(y-x)z+y^2-x^2=12$,

or, $(y-x)z+(y+x)(y-x)=12$,

and $(x+y+z)(y-x)=12$,

or $$x+y+z=\frac{12}{y-x}.$$

Hence, by equality, $$\frac{12}{z-y}=\frac{12}{y-x};$$

and, consequently, $z-y=y-x$, or $y=\dfrac{x+z}{2}$;

and hence, the numbers are in arithmetical proportion.

If, then, we denote their common difference by r, the numbers may be represented by

$$y-r, \quad y, \quad \text{and} \quad y+r.$$

The sum of the numbers will be now represented by $3y$. But we have seen, from the former equation, that the sum is equal to 12 divided by the common difference. Hence,

$$3y=\frac{12}{r}, \quad \text{or} \quad y=\frac{4}{r}, \quad \text{or} \quad r^2=\frac{16}{y^2}.$$

If, now, we substitute the new representatives of the numbers in equation (2.), we have

$$y^2-r^2+y^2-2ry+r^2+y^2+2ry+r^2=49\,;$$

that is, $$3y^2+r^2=49\,;$$

and substituting for r^2, its value, we have

$$3y^2+\frac{16}{y^2}=49,$$

or, $$y^4-\frac{49}{3}y^2=-\frac{16}{3}\,;$$

putting $z=y^2$, it becomes,

$$z^2-\frac{49}{3}z=-\frac{16}{3},$$

from which we find $z=16$,

and, consequently, $y=4$. Hence,

the numbers are 3, 4, and 5.

14. Find two numbers, such, that their difference, added to the difference of their squares, shall be equal to 150, and whose sum added to the sum of their squares, shall be equal to 330

Let the larger number be denoted by x, and the smaller by y. Then,

by 1st condition, $x-y+x^2-y^2=150$,

by 2d condition, $x+y+x^2+y^2=330$;

and by adding the equations, we have,

$$2x^2+2x=480,$$

and, $$x^2+x=240,$$

which gives, $x=15$, and $y=9$.

15. It is required to find a number consisting of three digits, such, that the sum of the squares of the digits shall be 104; the square of the middle digit exceeds twice the product of the other two by 4; and if 594 be subtracted from the number, the three digits become inverted.

Let $x=$ the left-hand digit, $y=$ the middle, and $z=$ the right-hand digit. Then,

$$x^2+y^2+z^2=104, \quad (1.)$$

$$y^2=2xz+4, \quad (2.)$$

and $$100x+10y+z-594=100z+10y+x. \quad (3.)$$

If we substitute for y^2 in equation (1.), its value in the second equation, we have, after transposing the 4,

$$x^2+2xz+z^2=100,$$

and, since both members are perfect squares,

$$x+z=10;$$

after which we can easily find the values of the unknown quantities; viz.: $x=8$, $y=6$, $z=2$.

It frequently happens that questions may be simplified by the introduction of an auxiliary unknown quantity into the statement of the question, as in the three following questions.

16. The sum of two numbers and the sum of their squares being given to find the numbers.

Let the sum of the numbers be denoted by $2a$, and the sum of their squares by $2b$; then $a=$ half the sum, and $b=$ half the sum of their squares; and let $x=$ half the difference of the numbers. Then,

$a+x=$ the greater; and $a-x=$ the less. Then, by squaring, and adding, we have

$$x^2+2ax+a^2+a^2-2ax+x^2=2b;$$

that is, $2x^2+2a^2=2b$, or $x=\sqrt{b-a^2}$.

Hence, the numbers are, $a+\sqrt{b-a^2}$, and $a-\sqrt{b-a^2}$.

17. The sum, and the sum of the cubes, of two numbers being given, to find the numbers.

Let the sum be denoted by $2a$, and the sum of their cubes by $2b$; and let $x=$ half their difference. Then, the numbers will be denoted by $a+x$, and $a-x$; and we shall have

$$(a+x)^3+(a-x)^3=c;$$

that is, by cubing and reducing,

$$2a^3+6ax^2=c.$$

Hence, $x^2=\frac{c-2a^3}{6a}$, and $x=\sqrt{\frac{c}{6a}-\frac{a^2}{3}}$

18. To find three numbers in arithmetical progression such, that their sum shall be equal to 18, and the product of he two extremes added to 25 shall be equal to the square of the mean

Let $x=$ the first number, and $y=$ the common difference
Then, $x, x+y$, and $x+2y$ will represent the numbers
Also, $3x+3y=18$, or $x+y=6$,

$$x^2+2xy+25=x^2+2xy+y^2,$$

or, $y^2=25$, and $y=5$
Whence, $x=1$, and the numbers are 1, 6, and 11.

19. Having given the sum, and the sum of the fourth powers of two numbers; to find the numbers.

Let the numbers be denoted as in the seventeenth question.
Then

$$(a+x)^4+(a-x)^4=2b;$$

which will give, after raising to the fourth powers, and reducing,

$$x^4+6a^2x^2=b-a^4,$$

and substituting z for x^2, we have,

$$z^2+6a^2z=b-a^4,$$

from which we have

$$z=-3a^2\pm\sqrt{b+8a^4};$$

and consequently, $x=\pm\sqrt{-3a^2\pm\sqrt{b+8a^4}}$.

20. To find three numbers in arithmetical progression, such, that the sum of their squares shall be equal to 1232, and the square of the mean greater than the product of the two extremes, by 16.

Let the mean be denoted by x, and the common difference by y; then the numbers will be represented by

$$x-y, \quad x, \quad \text{and} \quad x+y;$$

and by the conditions we shall have,

$$(x-y)^2+x^2+(x+y)^2=1232, \quad (1.)$$

and,
$$x^2=(x-y)\times(x+y)+16, \quad (2.)$$

By squaring the terms in the first equation, and reducing, we have

$$3x^2+2y^2=1232;$$

and by performing the multiplications, and reducing, in the second equation,

$$x^2=x^2-y^2+16;$$

that is,
$$y^2=16, \quad \text{and} \quad y=4.$$

Substituting this value for y^2 in the equation above, and we have

$$3x^2+32=1232,$$

and
$$x^2=400, \quad \text{or} \quad x=20.$$

Hence, the numbers are 16, 20, and 24.

21. To find two numbers whose sum is 80, and such, that if they be divided alternately by each other, the sum of the quotients shall be $3\frac{1}{3}$.

Let the sum $80=a$, and $b=3\frac{1}{3}$. Also, let one of the numbers be represented by x: then the other will be denoted by $a-x$; and we shall have

$$\frac{x}{a-x}+\frac{a-x}{x}=b.$$

Clearing the equation of fractions, we have

$$x^2+a^2-2ax+x^2=abx-bx^2,$$

that is,
$$2x^2+bx^2-2ax-abx=-a^2,$$

or
$$(2+b)x^2-(2+b)ax=-a^2$$

Hence, $$x^2-ax=-\frac{a^2}{2+b};$$

whence, $$x^2-ax+\frac{a^2}{4}=-\frac{a^2}{2+b}+\frac{a^2}{4},$$

and, $$x=\frac{a}{2}\pm\sqrt{\frac{a^2}{4}-\frac{a^2}{2+b}};$$

and substituting for a and b their values, we have

$$x=60, \quad \text{or} \quad x=20.$$

22 To find two numbers whose difference shall be 10, and if 600 be divided by each of them, the difference of the quotients shall also be 10.

Let the lesser number be denoted by x, and the greater will then be represented by $x+10$; and we shall have

$$\frac{600}{x}-\frac{600}{x+10}=10.$$

By clearing the fraction, we have

$$600x+6000-600x=10x^2+100x.$$

Hence, $$x^2+10x=600,$$

and, $x=20$, and $20+10=30$, the other number.

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